

Chapter 9: Space Trajectories

- The LANDSAT-4 orbit parameters are approximately $T = 99 \text{ min} = 5940 \text{ s}$, $r = 7,088,978.63292 \text{ m}$, and $v = 7,498.54653018 \text{ m/s}$. Using this information to set the viewing window and the initial conditions as in Examples 1 and 2, you get a circular orbit out a little further from the Earth's surface than the shuttle in Example 2.
- Starting at the circular orbit with the larger radius $r_2 = 7,859,745 \text{ m}$, you move to the smaller radius $r_1 = 6,711,505 \text{ m}$ by first applying

$$-\Delta v_2 = -286.345827452 \text{ m/s}$$

to leave the outer circular orbit. Then apply

$$-\Delta v_1 = -297.886857671 \text{ m/s}$$

at some time when the Hohmann transfer orbit touches the inner circular orbit.

- Assuming a circular orbit with radius $r = 3.844 \text{ E8 m}$, the period of such a lunar orbit would be $T = 2371843.4052 \text{ s} = 27.4518912639 \text{ days}$. (Alternatively, assuming a circular orbit with period $T = 27.321661 \text{ days} = 2360591.5104 \text{ s}$, you get a radius of $r = 383183321.767 \text{ m} \hat{=} 3.832 \text{ E5 km}$.) Since a circular model doesn't fit the observed values very well, try an elliptical model, namely a Hohmann transfer orbit between theoretical circular orbits at the extremes of the observed radii. Using $r_1 = 3.564 \text{ E8 m}$ and $r_2 = 4.067 \text{ E8 m}$, you get a transfer period of $T_h = 2345514.54703 \text{ s} = 27.1471591092 \text{ days}$. To plot this, one needs either the velocity at the *perifocal distance* (r_1 , the position on the orbit closest to the center of the earth) which will be

$$v_1 + \Delta v_1 = \sqrt{G M_e} \sqrt{\frac{2}{r_1} - \frac{2}{r_1 + r_2}} = 1091.84611706 \text{ m/s}$$

or the velocity at the *apofocal distance* (r_2 , the position on the orbit furthest from the center of the earth) which will be

$$v_2 - \Delta v_2 = \sqrt{G M_e} \sqrt{\frac{2}{r_2} - \frac{2}{r_1 + r_2}} = 956.808350432 \text{ m/s.}$$

- Stationary points on the x-axis satisfy

$$-\frac{GM}{X|X|} - \frac{Gm}{(X-D)|X-D|} + \omega^2 \left(X - \frac{mD}{M+m} \right) = 0.$$

Here you search for a root with $X < 0$, so both absolute-value terms can be replaced by negatives.

The equation to be solved reduces to the fifth-degree polynomial in X . Remembering

$$\omega^2 = \frac{G(M+m)}{D^3},$$

this gives

$$M(X - D)^2 + mX^2 + (M + m)X^2(X - D)^2 \left(X - \frac{mD}{M + m} \right) = 0.$$

Looking at the graph, you see a root approximately at $-D$. Using [SOLVER] on the TI-86 with a seed value $-D$ for X and bound = $\{-1 \text{ E}99, 0\}$, you find that $l_3 = -381675404.68362 \text{ m}$ (which is slightly greater than $-D = -384400000 \text{ m}$).

5. In a manner similar to Exercise 4, you search for a root with $0 < X < D$, so the terms are

$$|X| = X \quad \text{and} \quad |X - D| = -(X - D).$$

The equation to be solved reduces to the fifth-degree polynomial in X . Remembering

$$\omega^2 = \frac{G(M + m)}{D^3}$$

this gives

$$M(X - D)^2 - mX^2 - (M + m)X^2(X - D)^2 \left(X - \frac{mD}{M + m} \right) = 0.$$

Looking at the graph, you see a root approximately at $3.26 \text{ E}8$. Using [SOLVER] on the TI-86 with this seed value for X and bound = $\{0, D\}$, you find that $l_1 = 326380918.21025 \text{ m}$.

6. In a manner similar to Exercise 4, you search for a root with $D < X$, so the terms are

$$|X| = X \quad \text{and} \quad |X - D| = (X - D).$$

The equation to be solved reduces to the fifth-degree polynomial in X . Remembering

$$\omega^2 = \frac{G(M + m)}{D^3}$$

this gives

$$M(X - D)^2 - mX^2 - (M + m)X^2(X - D)^2 \left(X - \frac{mD}{M + m} \right) = 0.$$

Looking at the graph, you see a root approximately at $4.48 \text{ E}8$. Using [SOLVER] on the TI-86 with this seed value for X and bound = $\{D, 1 \text{ E}99\}$, you find that $l_2 = 448914836.87933 \text{ m}$.

7. Since the Sun-Jupiter ratio M/m is about $1.047 \text{ E}3$ (while the Earth-Moon ratio M/m is about $0.813 \text{ E}2$), Jupiter has less influence in this restricted three-body problem.
8. The trajectories you find are similar to the actual trajectories on space missions to the moon. The big difference is that an actual mission will make minor mid-course corrections, and will probably major Δv thrusts to move into and out of parking orbits near each large body.