Part A: Intersection of a line with a parabola

a. **<u>Algebra method</u>**

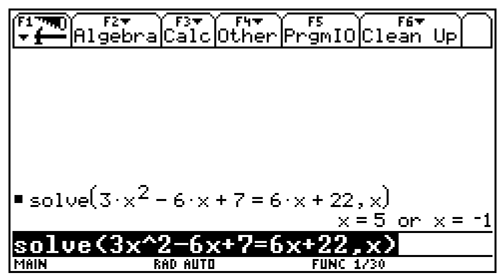
Use an algebraic method to find the points of intersection between the line with equation $y = 6x + 22$ and the parabola with equation $y = 3x^2 - 6x + 7$. Show how the 'discriminant' of an appropriate quadratic equation indicates that two solutions are expected and thus, that the graphs have two intersection points. <u>Technology check</u>: Use **Solve** to confirm your solutions.

$$3x^2 - 6x + 7 = 6x + 22$$
$$3x^2 - 12x - 15 = 0$$
$$3(x+1)(x-5) = 0$$
$$x = -1 \text{ or } 5$$
$$y = -6 + 22$$
$$= 16$$
$$y = 30 + 22$$
$$= 52$$

$$3x^2 - 12x - 15 = 0$$
$$\Delta = b^2 - 4ac$$
$$= (-12)^2 - 4 \times 3 \times -15$$
$$= 144 + 240$$
$$= 384$$

two solutions as discriminant is positive

Solve($3x^2 - 6x + 7 = 6x + 22, x$ **) enter**



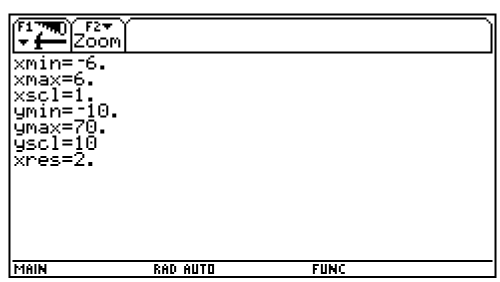
thus <u>points of intersection</u> are $(-1,16)$ and $(5,52)$

<u>Graphing Method</u>

Using an appropriate **Window**, draw the graphs on the calculator and confirm intersection points using **Intersection**. State the *x* and *y* values used for the graph window.

♦**Window**

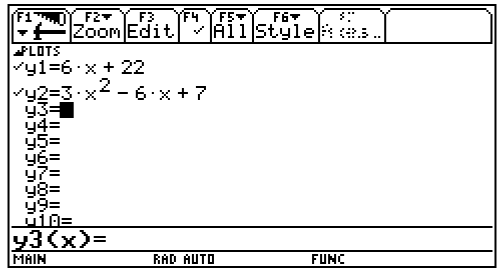
$x\min = -6$
$x\max = 6$
$x\text{scl} = 1$
$y\min = -10$
$y\max = 70$
$y\text{scl} = 10$



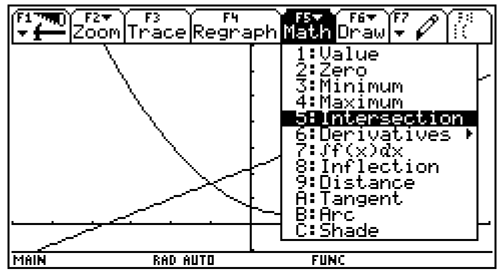
♦ **Y** =

$y1 = 6x + 22$

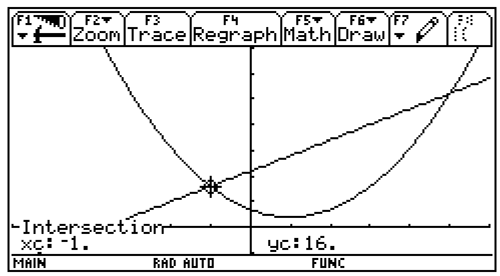
$y2 = 3x^2 - 6x + 7$



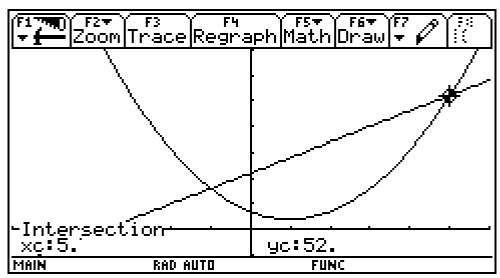
♦**Graph**



F5; 5(Intersection); enter; enter; Lower bound $x = -2$ **;**
Upper bound $x = 0$ **; enter**
Intersection point is $(-1,16)$

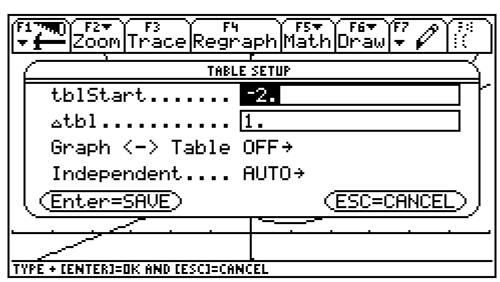


F5; 5(Intersection); enter; enter; Lower bound $x = 0$ **;**
Upper bound $x = 6$ **; enter**
Intersection point is $(5,52)$



Use the **Table** of values from the calculator to further confirm results.

♦**TblSet**
TblStart……. –2
Δ tbl………..1
enter;enter



♦ **Table**
Identify points of intersection

x	y1	y2
-2.	10.	31.
-1.	16.	16.
0.	22.	7.
1.	28.	4.
2.	34.	7.
3.	40.	16.
4.	46.	31.
5.	52.	52.

x=⁻1.

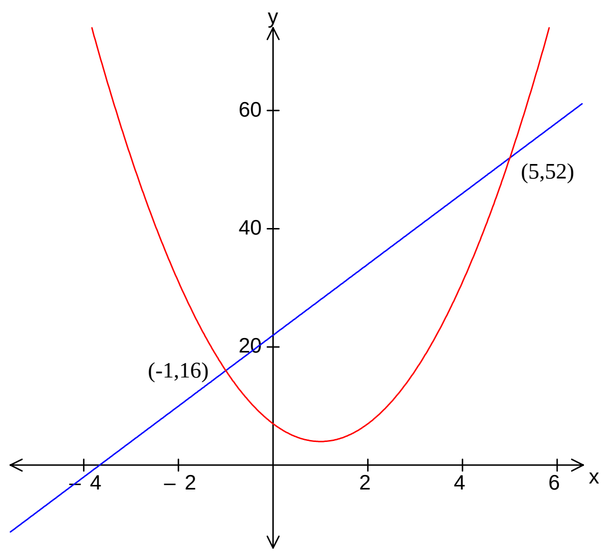
MAIN RAD AUTO FUNC

x	y1	y2
-2.	10.	31.
-1.	16.	16.
0.	22.	7.
1.	28.	4.
2.	34.	7.
3.	40.	16.
4.	46.	31.
5.	52.	52.

x=5.

MAIN RAD AUTO FUNC

Sketch the graph of each equation on the same set of axes and mark the points of intersection. With the graphs, display an appropriate table of values that show intersection points, as found from the calculator.



See above for table of values

b. Perform the same procedures as a. to find and confirm:

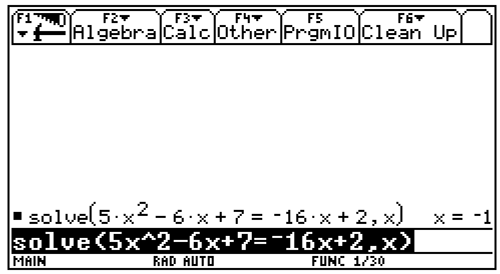
i. one point of intersection between the line with equation $y = -16x + 2$ and the parabola with equation $y = 5x^2 - 6x + 7$.

$$5x^2 - 6x + 7 = -16x + 2$$
$$5x^2 + 10x + 5 = 0$$
$$5(x+1)(x+1) = 0$$
$$5(x+1)^2 = 0$$
$$x = -1$$
$$y = 18$$

$$5x^2 + 10x + 5 = 0$$
$$\Delta = b^2 - 4ac$$
$$= 10^2 - 4 \times 5 \times 5$$
$$= 100 - 100$$
$$= 0$$

one solution as discriminant is zero

Solve($5x^2 - 6x + 7 = -16x + 2, x$ **) enter**



♦ **Window**

$x\min = -8$

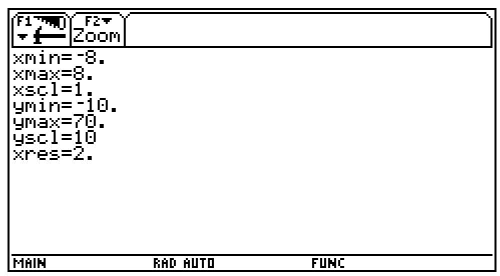
$x\max = 8$

$x\text{scl} = 1$

$y\min = -10$

$y\max = 70$

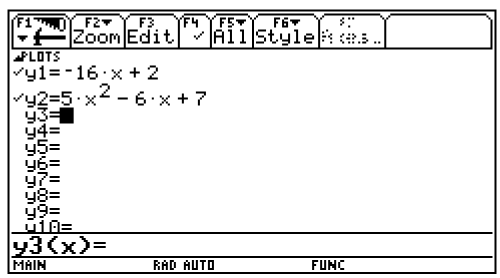
$y\text{scl} = 10$



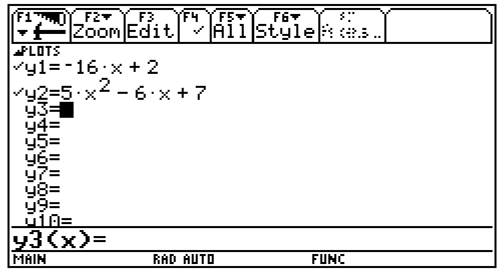
♦ **Y** =

$y1 = -16x + 2$

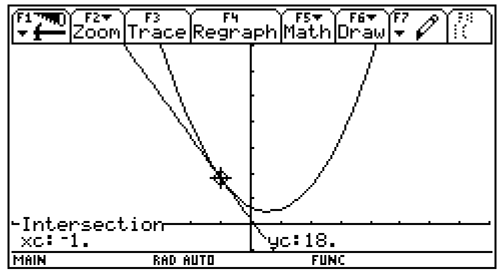
$y2 = 5x^2 - 6x + 7$



♦**Graph**

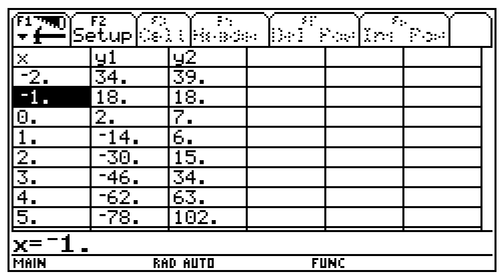


F5; 5(Intersection); enter; enter; Lower bound $x = -2$ **; Upper bound** $x = 0$ **; enter**
Intersection point is $(-1,18)$

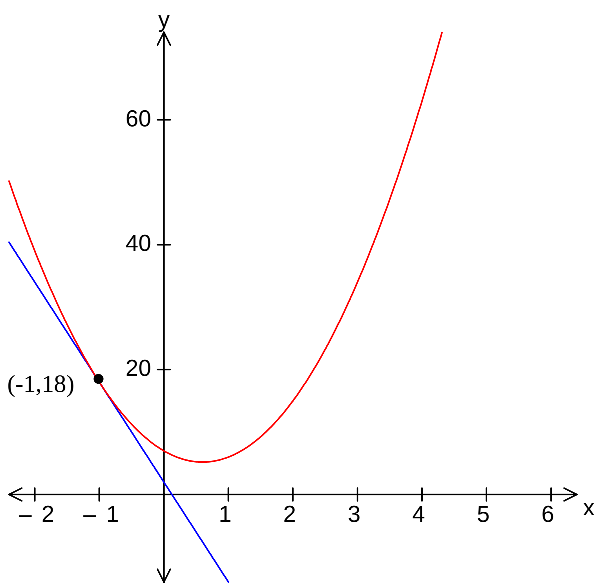


♦**TblSet**
TblStart……. –2
Δ tbl………..1
enter;enter

♦**Table**
Identify point of intersection



x	y1	y2
-2.	34.	39.
-1.	18.	18.
0.	2.	7.
1.	-14.	6.
2.	-30.	15.
3.	-46.	34.
4.	-62.	63.
5.	-78.	102.



See above for solutions

ii. no intersection points between the line with equation $y = 2x - 4$ and the parabola $y = 3x^2 - 6x + 7$.

$3x^2 - 6x + 7 = 2x - 4$

$3x^2 - 8x + 11 = 0$

no real solutions

no points of intersection

$3x^2 - 8x + 11 = 0$

$\Delta = b^2 - 4ac$

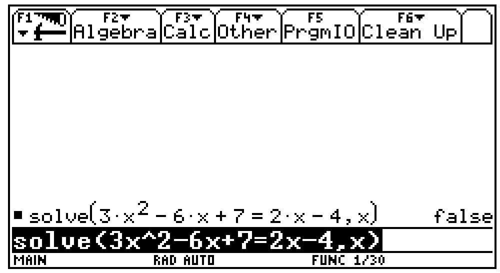
$= (-8)^2 - 4 \times 3 \times 11$

$= 64 - 132$

$= -64$

no solutions as discriminant is negative

Solve($3x^2 - 6x + 7 = 2x - 4, x$ **) enter**



♦ **Window**

$x\min = -8$

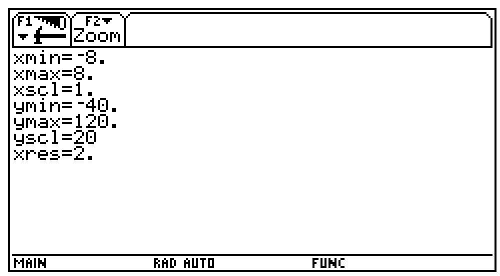
$x\max = 8$

$x\text{scl} = 1$

$y\min = -40$

$y\max = 120$

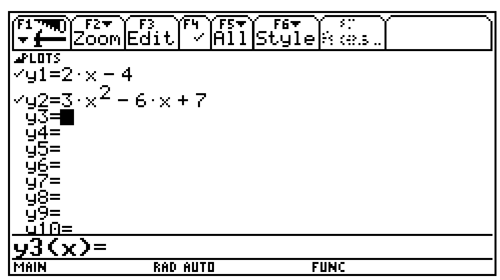
$y\text{scl} = 20$



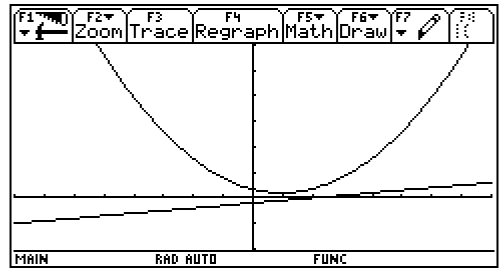
♦ **Y** =

$y1 = 2x - 4$

$y2 = 3x^2 - 6x + 7$

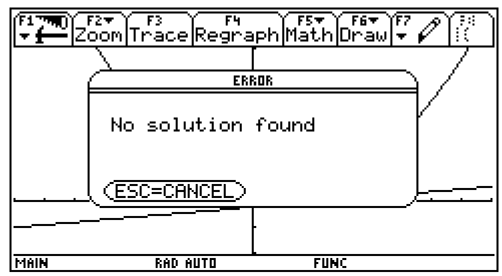


♦ **Graph**



F5; 5(Intersection); enter; enter; Lower bound $x = -8$ **; Upper bound** $x = 8$ **; enter**

No solution found



♦ **TblSet**

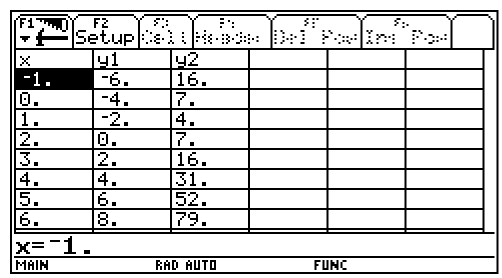
TblStart……. –2

Δ tbl………..1

enter; enter

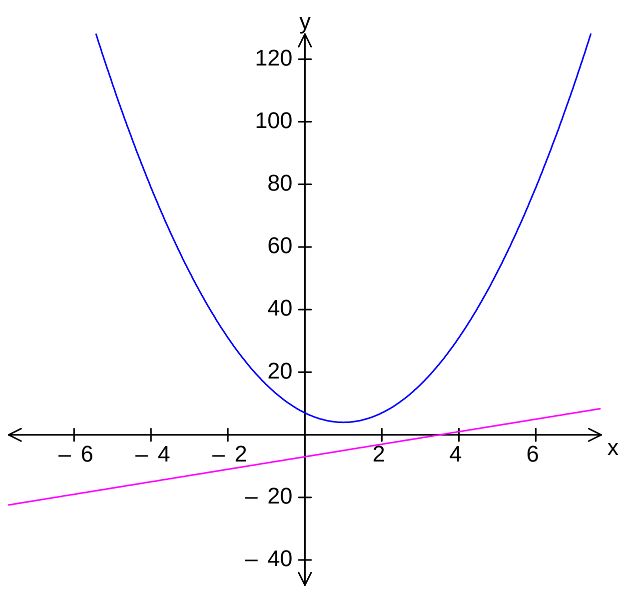
♦ **Table**

No intersection points



x	y1	y2
-1.	-6.	16.
0.	-4.	7.
1.	-2.	4.
2.	0.	7.
3.	2.	16.
4.	4.	31.
5.	6.	52.
6.	8.	79.

x=-1.



See above for table of values

c. Summarise the findings in an appropriate table, including how the 'discriminant' indicates the number of intersection points between the parabola and the line.

Line	Parabola	Quadratic equation for points of intersection	Discriminant	Points of intersection
$y=6x+22$	$y=3x^2-6x+7$	$3x^2-12x-15=0$	$\Delta=324$ positive $\to$ 2 sols	$(-1,16),(5,52)$
$y=-16x+2$	$y=5x^2-6x+7$	$5x^2+10x+5=0$	$\Delta=0$ zero $\to$ 1 sols	$(-1,18)$
$y=2x-4$	$y=3x^2-6x+7$	$3x^2-8x+11=0$	$\Delta=-20$ negative $\to$ 0 sols	No real solutions

Part B: Intersection of a line with a circle

a. **<u>Algebra method</u>**

Use an algebraic method to find the points of intersection between the line with equation $y=2x+4$ and the circle with equation $x^2+y^2=9$. Show how the 'discriminant' of an appropriate quadratic equation indicates that two solutions are expected and thus, that the graphs have two intersection points.
<u>Technology check</u>: Use **Solve** to confirm your solutions.

$x^2+(2x+4)^2=9$

$5x^2+16x+7=0$

$x=\dfrac{-16\pm\sqrt{(16)^2-4\times5\times7}}{2\times5}$

$=\dfrac{-8\pm\sqrt{29}}{5}$

substitute $x=\dfrac{-8\pm\sqrt{29}}{5}$ in $y=2x+4$

solutions are

$\left(\dfrac{-8+\sqrt{29}}{5},\dfrac{4+2\sqrt{29}}{5}\right),\left(\dfrac{-8-\sqrt{29}}{5},\dfrac{4-2\sqrt{29}}{5}\right)$

$5x^2+16x+16=0$

$\Delta=b^2-4ac$

$=(16)^2-4\times5\times7$

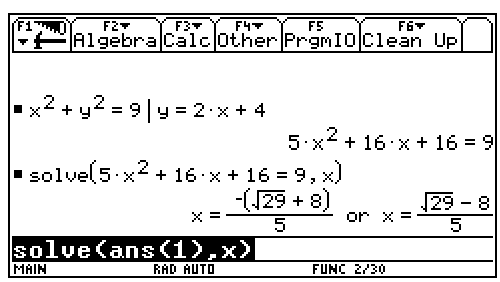
$=256-140$

$=116$

two solutions as discriminant is positive

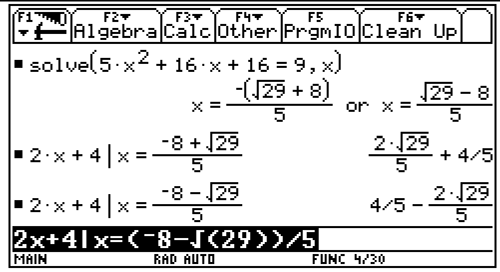
$x^2+y^2=9|y=2x+4$

Solve($5x^2+16x+16=9,x$ **) enter**



$2x+4\,\Big|\,x=\dfrac{-8+\sqrt{29}}{5}$

$2x+4\,\Big|\,x=\dfrac{-8-\sqrt{29}}{5}$



thus <u>points of intersection</u> are $\left(\frac{-8+\sqrt{29}}{5},\frac{4+2\sqrt{29}}{5}\right),\left(\frac{-8-\sqrt{29}}{5},\frac{4-2\sqrt{29}}{5}\right)$.

<u>Graphing method</u>

Using an appropriate **Window**, draw the graphs on the calculator and confirm intersection points using **Intersection**. State the x and y values used for the graph window.

♦ **Window**

$x\min = -4$

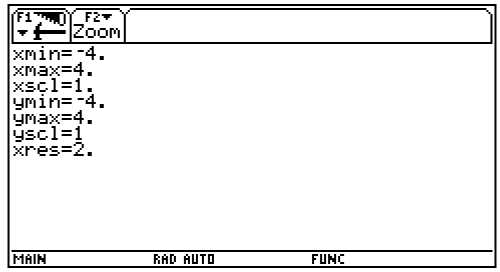
$x\max = 4$

$x\text{scl} = 1$

$y\min = -4$

$y\max = 4$

$y\text{scl} = 1$

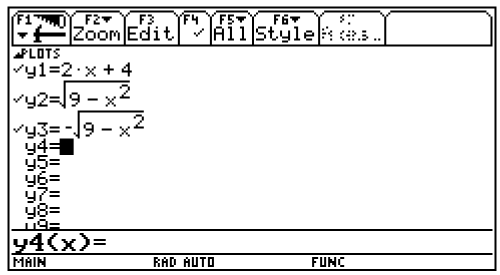


♦ **Y** =

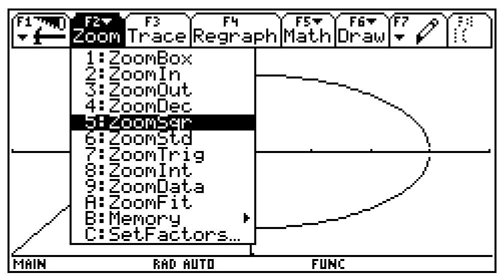
$y1 = 2x = 4$

$y2 = \sqrt{9 - x^2}$

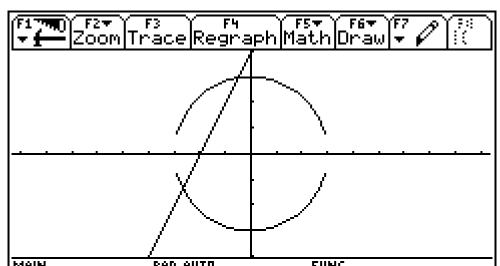
$y3 = -\sqrt{9 - x^2}$



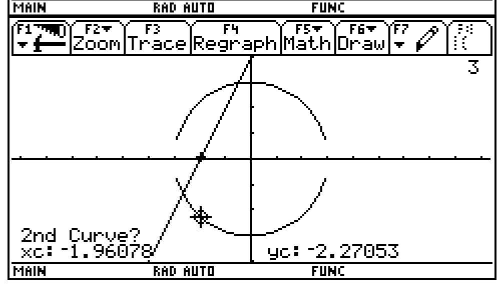
♦**Graph; F2, 5,**



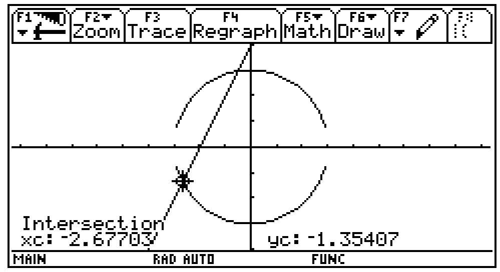
Enter



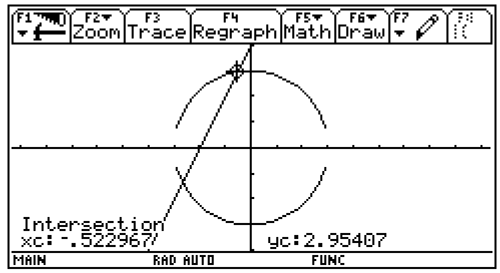
F5; 5(Intersection); enter; cursor to third curve; enter;



Lower bound $x = -3$ **; Upper bound** $x = -2$ **; enter**
(gives decimal approximation)

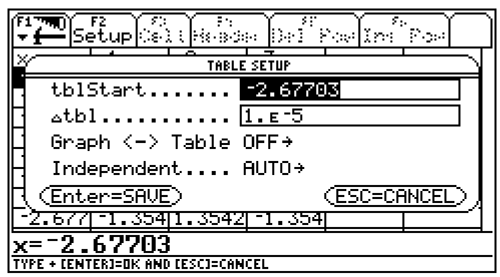


F5; 5(Intersection); enter; enter; Lower bound $x = -2$ **;**
Upper bound $x = 0$ **; enter**
(gives decimal approximation)

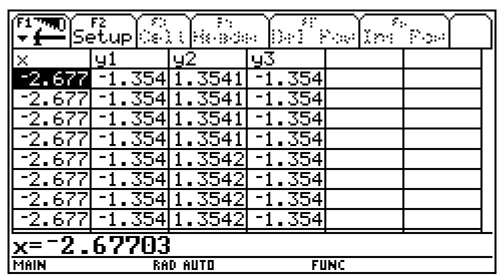


The **Table** <u>of values</u> from the calculator can only confirm approximate solutions.

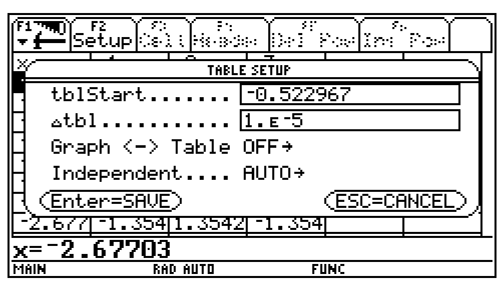
♦ **TblSet**
TblStart……. –2.67703
Δ tbl………..0.00001
enter;enter



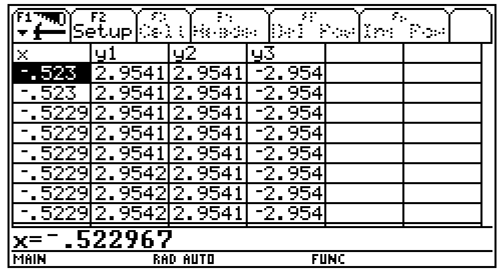
♦ **Table**
Identify points of intersection



♦ **TblSet**
TblStart……. –0.0522967
Δ tbl………..0.00001
enter;enter



♦ **Table**
Identify points of intersection



x	y1	y2	y3
-.523	2.9541	2.9541	-2.954
-.523	2.9541	2.9541	-2.954
-.5229	2.9541	2.9541	-2.954
-.5229	2.9541	2.9541	-2.954
-.5229	2.9541	2.9541	-2.954
-.5229	2.9542	2.9541	-2.954
-.5229	2.9542	2.9541	-2.954
-.5229	2.9542	2.9541	-2.954

x= -.522967

MAIN RAD AUTO FUNC

Sketch the graph of each equation on the same set of axes and mark the points of intersection. With the graphs, display an appropriate table of values that show intersection points, as found from the calculator.

b. Perform the same procedures as a. to find and confirm:

i. one point of intersection between the line with equation $y = x + 6$ and the circle with equation $x^2 + y^2 = 18$.

ii. no intersection points between the line with equation $y = x - 4$ and the circle with equation $(x-1)^2 + (y-2)^2 = 9$.

c. Summarise the findings in an appropriate table, including how the ‘discriminant’ indicates the number of intersection points between the parabola and the line.

Part C: Intersection of a straight line with a rectangular hyperbola

a. Use an algebraic method to find the points of intersection between the line with equation $y = 4x - 7$ and the hyperbola with equation $y = \frac{1}{x-2}$. Show how the 'discriminant' of an appropriate quadratic equation indicates that two solutions are expected and thus, that the graphs have two intersection points.
<u>Technology check</u>: Use **Solve** to confirm your solutions.

Using an appropriate **Window**, draw the graphs on the calculator and confirm intersection points using **Intersection**. State the *x* and *y* values used for the graph window.

Use the <u>**Table** of values</u> from the calculator to further confirm results.

Sketch the graph of each equation on the same set of axes and mark the points of intersection. With the graphs, display an appropriate table of values that show intersection points, as found from the calculator.

b. Perform the same procedures as a. to find and confirm:
   i. one point of intersection between the line with equation $x = 6$ and the hyperbola with equation $y = \frac{1}{x-4}$.
   ii. no intersection points between the line with equation $y = -x - 4$ and the hyperbola with equation $y = \frac{1}{x+3}$.

c. Summarise the findings in an appropriate table, including how the 'discriminant' indicates the number of intersection points between the parabola and the line.

Part D: General Case for the points of intersection between a line and a parabola

a. Use an algebraic method to find the points of intersection between the line with equation $y = px + q$ and the parabola with equation $y = ax^2 + bx + c$, where $a = 1$. Show that the 'discriminant' of an appropriate quadratic equation to determine the number of points of intersection is $\Delta = (b - p)^2 - 4(c - q)$.

b. If $b = 3$; $c = 2$ and $p = 1$, find the values of *q* that would allow i. one point of intersection; ii. two points of intersection; iii. no points of intersection. Give an example of each.
<u>Technology check</u>: Use **Solve** to confirm your solutions.

For i. and ii., use an appropriate **Window**, draw the graphs on the calculator and confirm intersection points using **Intersection**.

Sketch the graph of each equation on the same set of axes and mark the points of intersection. With the graphs, display an appropriate table of values that show intersection points, as found from the calculator.

c. If $b = 3$; $c = 2$ and $q = 1$, find the values of *p* that would allow i. one point of intersection; ii. two points of intersection; iii. no points of intersection. Give an example of each.
<u>Technology check</u>: Use **Solve** to confirm your solutions.

For i. and ii., use an appropriate **Window**, draw the graphs on the calculator and confirm intersection points using **Intersection**.

Sketch the graph of each equation on the same set of axes and mark the points of intersection. With the graphs, display an appropriate table of values that show intersection points, as found from the calculator.

d. Give a brief discussion of the findings between Part D, b. and c.