### Part A: Intersection of a line with a parabola

#### a. <u>Algebra method</u>

Use an algebraic method to find the points of intersection between the line with equation y = 6x + 22 and the parabola with equation  $y = 3x^2 - 6x + 7$ . Show how the 'discriminant' of an appropriate quadratic

equation indicates that two solutions are expected and thus, that the graphs have two intersection points. <u>Technology check</u>: Use **Solve** to confirm your solutions.

$$3x^{2} - 6x + 7 = 6x + 22$$
  

$$3x^{2} - 12x - 15 = 0$$
  

$$3(x+1)(x-5) = 0$$
  

$$x = -1 \text{ or } 5$$
  

$$y = -6 + 22$$
  

$$= 16$$
  

$$y = 30 + 22$$
  

$$= 52$$

$$3x^{2} - 12x - 15 = 0$$

$$\Delta = b^{2} - 4ac$$

$$= (-12)^{2} - 4 \times 3 \times -15$$

$$= 144 + 240$$

$$= 384$$
two solutions of discriminant is not

two solutions as discriminant is positive

**Solve**
$$(3x^2 - 6x + 7 = 6x + 22, x)$$
 enter

	Algebra	Calc	Other	PrgmIO	Clear	ΰUp	
∎ sol•	ve(3·x²	- 6 · ×	+ 7 = 6	·×+22	, <u>x</u> )		
-				X	= 5 or	<u> </u>	-1
solv	ve(3x^	<u>2-6&gt;</u>	(+7=6	x+22,	.x)		
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/24.7 P

thus <u>points of intersection</u> are (-1,16) and (5,52)

### **Graphing Method**

Using an appropriate **Window**, draw the graphs on the calculator and confirm intersection points using **Intersection**. State the *x* and *y* values used for the graph window.

#### ♦ Window

 $x \min = -6$   $x \max = 6$   $x \operatorname{scl} = 1$   $y \min = -10$   $y \max = 70$  $y \operatorname{scl} = 10$ 

F170 F27 ZOOM			
xmin=-6. xmax=6.			
xscl=1. ymin=-10.			
yscl=10 xres=2.			
MAIN	RAD AUTO	FUNC	



Use the **<u>Table</u>** of values from the calculator to further confirm results.

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# ♦ Table

Identify points of intersection

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-1.	16.	16.			
0.	22.	7.			
1.	28.	4.			
2.	34.	7.			
3.	40.	16.			
4.	46.	31.			
5.	52.	52.			
x=-1	•				
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F1 7 € × -2. -1. 0. 1. 2. 3.	F2 Setup 10. 16. 22. 28. 34. 40.	31 92 31. 16. 7. 4. 7. 16.		Poe Ins	
€1     0       -1.     0.       1.     2.       3.     4.	F2 Setup 10. 16. 22. 28. 34. 40.	9 92 31. 16. 7. 4. 7. 16. 31.			
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F <sup>1</sup> 770 × -2. -1. 0. 1. 2. 3. 4. 5. x=5.	F2 5: etup ::= 10. 16. 22. 28. 34. 40. 46. 52.	92 31. 16. 7. 4. 7. 16. 31. 52.			

Sketch the graph of each equation on the same set of axes and mark the points of intersection. With the graphs, display an appropriate table of values that show intersection points, as found from the calculator.



b. Perform the same procedures as a. to find and confirm:

one point of intersection between the line with equation y = -16x + 2 and the parabola

with equation 
$$y = 5x^2 - 6x + 7$$
. $5x^2 - 6x + 7 = -16x + 2$  $5x^2 + 10x + 5 = 0$  $5x^2 + 10x + 5 = 0$  $\Delta = b^2 - 4ac$  $5(x+1)(x+1) = 0$  $= 10^2 - 4 \times 5 \times 5$  $5(x+1)^2 = 0$  $= 100 - 100$  $x = -1$  $= 0$  $y = 18$ one solution as discriminant is zero

**Solve** $(5x^2 - 6x + 7 = -16x + 2, x)$  enter

i.

[ <mark>-1790</mark> ] F2 	braCalcOthe	r <mark>F5 F6</mark> rPrgmIOClean	Up
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 F2

 Zoom
 xmax=8.

 xmax=8.
 xscl=1.

 ymin=-10.
 ymax=70.

 yscl=10
 xres=2.

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 F1700
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♦ Window

 $x \min = -8$   $x \max = 8$   $x \operatorname{scl} = 1$   $y \min = -10$   $y \max = 70$  $y \operatorname{scl} = 10$ 

♦ Y =

$$y1 = -16x + 2$$
$$y2 = 5x^2 - 6x + 7$$

### ♦ Graph





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x y1 y2	
-2. 34. 39.	
-1. 18. 18.	
0. 2. 7.	
114. 6.	
230. 15.	
346. 34.	
462. 63.	
578. 102.	
x=-1.	
MAIN RAD AUTO FUNC	

**F5; 5(Intersection); enter; enter; Lower bound** x = -2; **Upper bound** x = 0; **enter** Intersection point is (-1,18)

# ♦ TblSet

TblStart......-2  $\Delta$  tbl......1 enter;enter

### ♦ Table

Identify point of intersection

See above for solutions



ii. no intersection points between the line with equation y = 2x - 4 and the parabola  $y = 3x^2 - 6x + 7$ .

$$3x^{2}-6x+7 = 2x-4$$

$$3x^{2}-8x+11 = 0$$

$$3x^{2}-8x+11 = 0$$

$$\Delta = b^{2}-4ac$$

$$= (-8)^{2}-4\times 3\times 11$$

$$= 64-132$$

$$= -64$$

**Solve** $(3x^2 - 6x + 7 = 2x - 4, x)$  enter

no solutions as discriminant is negative



xmin=-8. xmax=8.		
xscl=1. ymin=−40. umay=120		
yscl=20 xres=2.		
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⊿PLOTS ⊻y1=2·× -	- 4		
<ul> <li>✓y2=3·×<sup>2</sup></li> </ul>	- 6·×+7		
93 <b>-</b> ∎ 94= 95=			
90- 96= 97=			
98= 99=			
<u>u10=</u> u3(x)=			
MAIN	RAD AUTO	FUNC	

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$x \min = -8$
$x \max = 8$
xscl = 1
$y \min = -40$
$y \max = 120$
yscl = 20

 $\mathbf{Y} = y\mathbf{1} = 2x - 4$  $y\mathbf{2} = 3x^2 - 6x + 7$ 

# ♦ Graph





F <b>177</b> ▼∰S	F2 etup(s)		6 (3+1 <sup>37</sup> )	o In	* Post
X	y1	y2			
-1.	-6.	16.			
0.	-4.	7.			
1.	-2.	4.			
2.	0.	7.			
3.	2.	16.			
4.	4.	31.			
5.	6.	52.			
6.	8.	79.			
x=-1.					
MAIN	BA	D AUTO	FL	INC	

See above for table of values

**F5; 5(Intersection); enter; enter; Lower bound** x = -8; **Upper bound** x = 8; **enter** No solution found

# ♦ TblSet

TblStart......-2  $\Delta$  tbl......1 enter; enter

# ♦ Table

No intersection points



Line	Parabola	Quadratic equation	Discriminant	Points of
		for points of		intersection
		intersection		
y = 6x + 22	$y = 3x^2 - 6x + 7$	$3x^2 - 12x - 15 = 0$	$\Delta = 324$	(-1,16), (5,52)
	·		positive $\rightarrow 2$ sols	
y = -16x + 2	$y = 5x^2 - 6x + 7$	$5x^2 + 10x + 5 = 0$	$\Delta = 0$	(-1,18)
	2		$zero \rightarrow 1 sols$	
y = 2x - 4	$y = 3x^2 - 6x + 7$	$3x^2 - 8x + 11 = 0$	$\Delta = -20$	No real solutions
	•		negative $\rightarrow 0$ sols	

c. Summarise the findings in an appropriate table, including how the 'discriminant' indicates the number of intersection points between the parabola and the line.

### Part B: Intersection of a line with a circle

## a. <u>Algebra method</u>

Use an algebraic method to find the points of intersection between the line with equation y = 2x + 4 and the circle with equation  $x^2 + y^2 = 9$ . Show how the 'discriminant' of an appropriate quadratic equation indicates that two solutions are expected and thus, that the graphs have two intersection points. <u>Technology check</u>: Use **Solve** to confirm your solutions.

$$x^{2} + (2x+4)^{2} = 9$$

$$5x^{2} + 16x + 16 = 0$$

$$5x^{2} + 16x + 7 = 0$$

$$x = \frac{-16 \pm \sqrt{(16)^{2} - 4 \times 5 \times 7}}{2 \times 5}$$

$$= \frac{-8 \pm \sqrt{29}}{5}$$

$$x^{2} + y^{2} = 9|y = 2x + 4$$
solutions are
$$\left(\frac{-8 + \sqrt{29}}{5}, \frac{4 + 2\sqrt{29}}{5}\right), \left(\frac{-8 - \sqrt{29}}{5}, \frac{4 - 2\sqrt{29}}{5}\right)$$

$$x^{2} + y^{2} = 9|y = 2x + 4$$

$$solve(5x^{2} + 16x + 16 = 9, x) \text{ enter}$$

$$x^{2} + y^{2} = 9|y = 2x + 4$$

$$x^{2} + y^{2} = 9|y = 2x + 4$$

$$x^{2} + y^{2} = 9|y = 2x + 4$$

$$x^{2} + y^{2} = 9|y = 2x + 4$$

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$$x^{2} + y^{2} = 9|y = 2 + 4$$

$$x^{2} + y^{2} = 9|y = 2 + 4$$

$$x^{2} + y^{2} = 9|y = 2 + 4$$

$$2x+4 \left| x = \frac{-8+\sqrt{29}}{5} \right|$$
$$2x+4 \left| x = \frac{-8-\sqrt{29}}{5} \right|$$

solve(ans(1).x)

 $=\frac{-(\sqrt{29}+8)}{5}$  or x

# Non-linear Simultaneous Equations Applicaton Task



thus points of intersection are 
$$\left(\frac{-8+\sqrt{29}}{5}, \frac{4+2\sqrt{29}}{5}\right), \left(\frac{-8-\sqrt{29}}{5}, \frac{4-2\sqrt{29}}{5}\right).$$

# **Graphing method**

Using an appropriate **Window**, draw the graphs on the calculator and confirm intersection points using **Intersection**. State the *x* and *y* values used for the graph window.

# ♦ Window

♦ Y =

y1 = 2x = 4

♦ Graph; F2, 5,

 $y2 = \sqrt{9 - x^2}$  $y3 = -\sqrt{9 - x^2}$ 

 $x \min = -4$  $x \max = 4$  $x \operatorname{scl} = 1$  $y \min = -4$  $y \max = 4$  $y \operatorname{scl} = 1$ 









Enter

F5; 5(Intersection); enter; cursor to third curve; enter;

**Lower bound** x = -3; **Upper bound** x = -2; **enter** (gives decimal approximation)



**F5; 5(Intersection); enter; enter; Lower bound** x = -2; **Upper bound** x = 0; **enter** (gives decimal approximation)



The  $\underline{\text{Table of values}}$  from the calculator can only confirm approximate solutions.

### ♦ TblSet

TblStart......-2.67703  $\Delta$  tbl.....0.00001 enter;enter

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x=-2.67703
TYPE + CENTER3=OK AND CESC3=CANCEL

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-2.677	-1.354	1.3541	-1.354					
-2.677	-1.354	1.3541	-1.354					
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-2.677	-1.354	1.3541	-1.354					
-2.677	-1.354	1.3542	-1.354					
-2.677	-1.354	1.3542	-1.354					
-2.677	-1.354	1.3542	-1.354					
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x=-2.67703								
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-2.677 -1.354 1.3542 -1.354	=1						
x=-2.67703							
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◆Table Identify points of intersection

# ♦ TblSet

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### ♦ Table Identify points of intersection

Fim Fi Setup (s) (History Del For Int For								
×	y1	y2	y3					
523	2.9541	2.9541	-2.954					
523	2.9541	2.9541	-2.954					
5229	2.9541	2.9541	-2.954					
5229	2.9541	2.9541	-2.954					
5229	2.9541	2.9541	-2.954					
5229	2.9542	2.9541	-2.954					
5229	2.9542	2.9541	-2.954					
5229	2.9542	2.9541	-2.954					
x=522967								
MAIN	RA	D AUTO	FL	INC				

Sketch the graph of each equation on the same set of axes and mark the points of intersection. With the graphs, display an appropriate table of values that show intersection points, as found from the calculator.

- b. Perform the same procedures as a. to find and confirm:
  - i. one point of intersection between the line with equation y = x + 6 and the circle with equation  $x^2 + y^2 = 18$ .
  - ii. no intersection points between the line with equation y = x 4 and the circle with equation  $(x-1)^2 + (y-2)^2 = 9$ .
- c. Summarise the findings in an appropriate table, including how the 'discriminant' indicates the number of intersection points between the parabola and the line.

### Part C: Intersection of a straight line with a rectangular hyperbola

a. Use an algebraic method to find the points of intersection between the line with equation y = 4x - 7 and

the hyperbola with equation  $y = \frac{1}{x-2}$ . Show how the 'discriminant' of an appropriate quadratic equation indicates that two solutions are expected and thus, that the graphs have two intersection points.

Technology check: Use Solve to confirm your solutions.

Using an appropriate **Window**, draw the graphs on the calculator and confirm intersection points using **Intersection**. State the *x* and *y* values used for the graph window.

Use the <u>Table of values</u> from the calculator to further confirm results.

Sketch the graph of each equation on the same set of axes and mark the points of intersection. With the graphs, display an appropriate table of values that show intersection points, as found from the calculator.

b. Perform the same procedures as a. to find and confirm:

i.

one point of intersection between the line with equation x = 6 and the hyperbola with 1

equation 
$$y = \frac{1}{x-4}$$

- ii. no intersection points between the line with equation y = -x 4 and the hyperbola with equation  $y = \frac{1}{x+3}$ .
- c. Summarise the findings in an appropriate table, including how the 'discriminant' indicates the number of intersection points between the parabola and the line.

### Part D: General Case for the points of intersection between a line and a parabola

- a. Use an algebraic method to find the points of intersection between the line with equation y = px + q and the parabola with equation  $y = ax^2 + bx + c$ , where a = 1. Show that the 'discriminant' of an appropriate quadratic equation to determine the number of points of intersection is  $\Delta = (b - p)^2 - 4(c - q)$ .
- b. If b = 3; c = 2 and p = 1, find the values of q that would allow i. one point of intersection; ii. two points of intersection; iii. no points of intersection. Give an example of each. Technology check: Use **Solve** to confirm your solutions.

For i. and ii., use an appropriate **Window**, draw the graphs on the calculator and confirm intersection points using **Intersection**.

Sketch the graph of each equation on the same set of axes and mark the points of intersection. With the graphs, display an appropriate table of values that show intersection points, as found from the calculator.

c. If b = 3; c = 2 and q = 1, find the values of p that would allow i. one point of intersection; ii. two points of intersection; iii. no points of intersection. Give an example of each. Technology check: Use **Solve** to confirm your solutions.

For i. and ii., use an appropriate **Window**, draw the graphs on the calculator and confirm intersection points using **Intersection**.

Sketch the graph of each equation on the same set of axes and mark the points of intersection. With the graphs, display an appropriate table of values that show intersection points, as found from the calculator.

d. Give a brief discussion of the findings between Part D, b. and c.