Ladder Corner

Teacher Notes & Answers

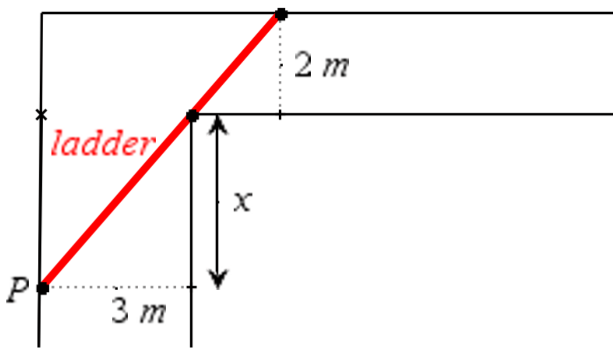
7 8 9 10 **11** **12**

TI-Nspire Investigation Student 20 min

Introduction

Maddi the painter has a selection of different length ladders. She has a job that requires her to paint some very high ceilings in a narrow corridor that contains several corners. Maddi would like to know what is the longest ladder she can take with her on this job.

She took some measurements of the corridors. One corridor has a width of 2 metres and the other a width of 3m as shown opposite.



Forming an Equation

Question: 1.

Maddi considers the first section of the ladder from point P to the corner. She refers to this section as L_1. Define a function $L_1(x)$ in terms of x.

$$L_1(x) = \sqrt{x^2 + 9}$$

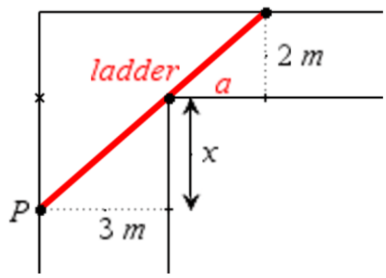
Question: 2.

Maddi notices that the second section of the ladder, which she calls L_2 (from corner to end) forms a similar triangle to the first section. Define a function $L_2(x)$ in terms of x for the second section of the ladder.

Let a be the side length of the triangle shown in the diagram below.

Using similar triangles: $\frac{3}{x} = \frac{a}{2}$

$$a = \frac{6}{x}$$



$$L_2(x) = \sqrt{4 + \frac{36}{x^2}}$$

Question: 3.

Define a function $L(x)$ in terms of x for the total length of the ladder including any domain restrictions.

$$L(x) = \sqrt{x^2 + 9} + \sqrt{4 + \frac{36}{x^2}} \text{ where } x > 0.$$

Note that the domain restriction for the expression is $x \in R \setminus \{0\}$.

The function $L(x)$ as it relates to the length of the ladder must assume the domain restriction $x > 0$ as the ladder could be infinitely long if $x < 0$.

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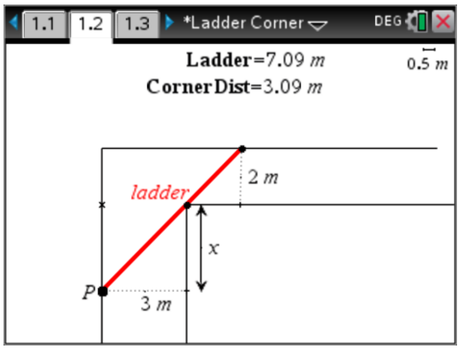
Validating the Equation

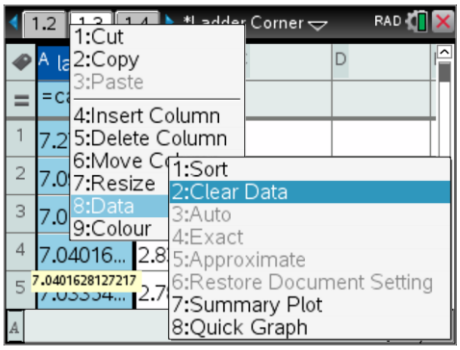


Open the TI-Nspire file "Ladder Corner".

Navigate to page 1.2

Grab point P and move it up and down. As point P moves up and down the height and area of the triangle is being collected automatically and stored in a spreadsheet on page 1.3.

Navigate to page 1.4 and graph the function $a(x)$ and confirm that it passes through the data points generated.

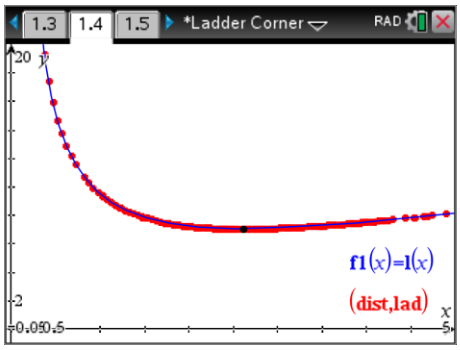


Teacher Notes:

The data is collected on Page 1.3 using an 'auto data capture' feature. To clear the data from these lists, navigate to the top of the list so that the entire column is highlighted then select:

Data > Clear Data

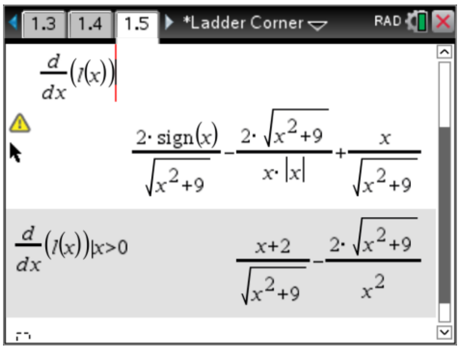
The data is automatically graphed on page 1.4. Students may wish to add a calculator application to define the function for the length of the ladder.



The function should perfectly match the data that has been collected from the diagram.

Question: 4.

Use CAS to determine the derivative of the function: $L(x)$.



$$\frac{d(L(x))}{dx} = \frac{x+2}{\sqrt{x^2+9}} - \frac{2\sqrt{x^2+9}}{x^2}$$

Teacher Notes:

There are several noteworthy items here. In the absence of any domain restrictions the derivative function includes:

- Sign(x)
- $|x|$

It is worth graphing the original function for $L(x)$ and both derivative functions so that students can see the difference. If students do the derivative by-hand they may simply gloss over the difference, with the graph in the first quadrant only they may never pick up the problem or significance of the signum or absolute functions.

With the domain restrictions in place students may also express the answer as: $L(x) = \dfrac{x^3 - 18}{x^2\sqrt{x^2+9}}$

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Question: 5.

Determine the minimum value of the function $L(x)$ and explain why this is the maximum length of the ladder.

Minimum value occurs when: $x = \sqrt[3]{18}$ or $x = 3^{\frac{2}{3}}2^{\frac{1}{3}}$ resulting in a maximum ladder length of approximately 7.0235m.

This is the maximum length of the ladder. If a larger ladder is taken into the corridors it will not fit part of the corner. Maddi and her painting crew would be stuck approximately 2.6m from the corner.

Question: 6.

Explain practically how Maddi might be able to get a longer ladder around the corner in the hallway.

This problem only considers the two dimensional situation. If the ceiling height in the building is approximately 2.7 metres then the ladder could be a little longer … approximately 7.5m. However in considering the three dimensional problem students would also have to consider the ladder as having width making the problem considerably more complicated.

Teacher Notes:

The "Extension" to the "Ladder problem" … worthy of inclusion for the pun alone, is to consider the general case where the two hallways are a units and b units wide. This results in a maximum ladder length of:

$$\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

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