

Sum Rectangles

ID: 12100

Time Required

15 minutes




Activity Overview

In this activity, students will graphically and numerically explore Riemann sums and develop an understanding of summation notation for adding these rectangles.

Topic: Riemann sums

- Graphically exploring Riemann sums
- Summation notation

Teacher Preparation and Notes

- The file *area.tns* is a library file. It contains several programs using Riemann sums, the Trapezoidal Rule, and Simpson's Rule. Once this file *area.tns* is saved in the **MyLib** folder, the programs within it can be accessed other TI-Nspire documents. Instructions on using the programs appear in the TI-Nspire document.
- If using the TI-Nspire software, put the library file *area.tns* in the folder `C:\MyDocuments\TI-Nspire\MyLib`.
- Students will write their responses directly into the TI-Nspire document and/or on the accompanying handout. On self-check questions, after answering the question students can press  and select **Check Answer** (or press  + ).
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- **To download the student TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "12100" in the keyword search box.**

Associated Materials

- *SumRectangles_Student.doc*
- *area.tns*
- *SumRectangles.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

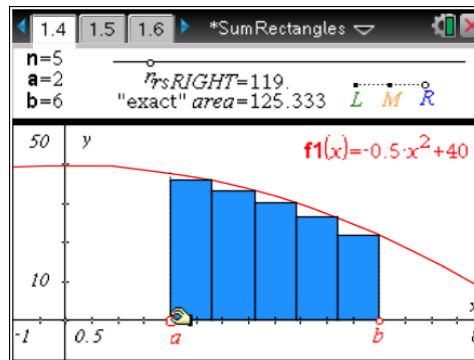
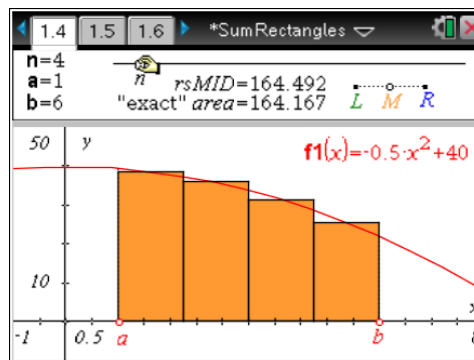
- *Riemann Rectangle Errors (TI-Nspire technology)* — 16107
- *Riemann Sums (TI-Nspire technology)* — 16102
- *Exploring the Area Under a Curve (TI-Nspire CAS technology)* — 8268
- *Riemann's Sums (TI-Nspire CAS technology)* — 9578

Part 1 – Graphical Riemann Sums

Students will grab point n on the slider on page 1.4 to change the number of subintervals. Many questions can be asked and observations made. This section can be extended by asking about other functions. The self-check questions instruct students to graph other functions that have different concavity or slope, but fit nicely in the window.

The *L M R* that is toward the right on the top application allows students toggle between left endpoint, midpoint and right endpoint Riemann sums.

Students get a visual of what Riemann sums are. If only a few rectangles are graphed students may be able to better see and understand why the midpoint is an over approximation in some situations and an under approximation at other times.



TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 1 at the end of this lesson.

Student Solutions

1. The midpoint approximation is between the left and right endpoint approximations. The left is too big and the right too small.
2. The Riemann sums converge (to the definite integral) as $n \rightarrow \infty$. With thinner widths, the rectangles approach the true area.
3. (a) over, (b) under, (c) over, (d) under. For example, when the function is decreasing and concave down, the function curves more steeply for the second half of the rectangle than the first. Therefore, since there is more rectangle above than below the graph, it over estimates the area.

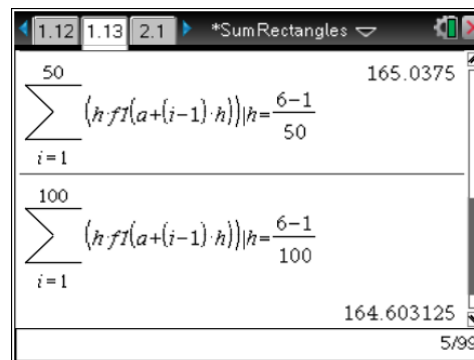
TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Part 2 – Summation notation

In Part 2, students examine the summation notation. By reading and interacting with the TI-Nspire document and answering questions on the student worksheet, students will better understand the sigma notation.

On page 1.10 students are to move the cursor over a math box (a dashed box will appear around the item when in the desired location) and select **MENU > Actions > Evaluate** to execute a command. This is why the expression $f1(a) + f1(a + 1) + \dots$ is repeated. When the second one is evaluated, the first will remain to show what was evaluated.



Student Solutions

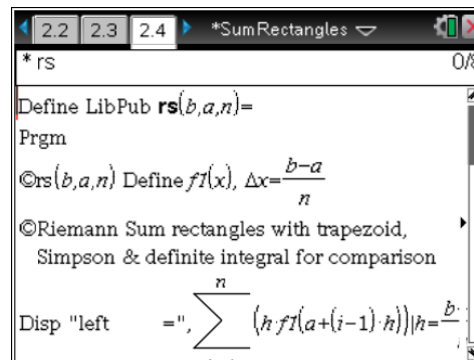
1. $\Delta x = h = 1$
2. $f1(a) + f1(a + 1) + f1(a + 2) + f1(a + 3) + f1(a + 4)$
3. The right Riemann sums from a to b with 5 subintervals, looks like $f1(a + 1) + f1(a + 2) + f1(a + 3) + f1(a + 4) + f1(a + 5)$. Note $a + 5 = b$ since $\Delta x = 1$
4. $\sum_{i=1}^n (h \cdot f1(a + (i - 1)h)) | h = \frac{6-1}{n}$

When $n = 10$, Left Riemann sum = 168.4375.
 When $n = 20$, Left Riemann sum = 166.328125
 When $n = 50$, Left Riemann sum = 165.0375
 When $n = 100$, Left Riemann sum = 164.603125

Extension – Area Programs

Students will use the programs to compare area approximation methods. One of the programs is opened so students can see that it is nothing more than the summation notation they had investigated in the previous part.

Have students open the file *area.tns* and put this file in the **MyLib** folder by pressing **doc** and selecting **File > Save As...** When prompted, select **MyLib** for the location to save and press **enter**.



TI-Nspire Navigator Opportunity: Send to Class
See Note 3 at the end of this lesson.

Then refresh the library to use these programs when you are in any document. To refresh the library, press **doc** and select **Refresh Libraries**.

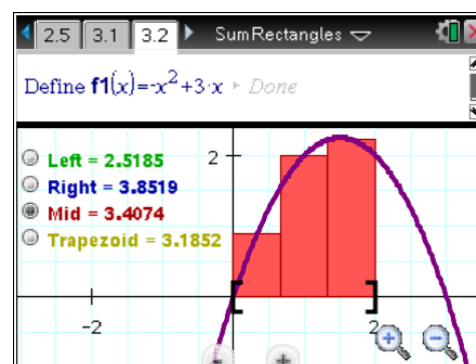
TI-Nspire Navigator Opportunity: Live Presenter**See Note 4 at the end of this lesson.****Student Solutions**

- | | | |
|---------------|-----------------|-----------------|
| $n = 10$ | $n = 100$ | $n = 500$ |
| mid = 71.5625 | mid = 71.665625 | mid = 71.666625 |
| trap = 71.875 | trap = 71.66875 | trap = 71.66675 |
- For $y = x^2$, an increasing concave up function in this domain, the midpoint is approaching $71\frac{2}{3}$ from the left and the trapezoid method is approaching the value of the definite integral from the right. Midpoint gives a better approximation. The program **rsa(b,a,n)** evaluates the area from a to b with n subintervals using various methods.

Extension #2 – Riemann Script Application

Students can use the Riemann Script application compare area approximation methods visually and numerically. The script provides an easy to use means of exploring left, right, midpoint and trapezoid approximations for $f_1(x)$. The function can be modified with the Notes application at the top of page 3.2. Note that the bottom application is not a Graphs application, so changing the size of the window will not behave as you are accustomed.

Use the – and + in the center bottom to decrease or add more subintervals. The + – magnifying glass will zoom in. Click the circle to pick which approximation you would like to see.

**TI-Nspire Navigator Opportunity: Screen Capture****See Note 4 at the end of this lesson.**

TI-Nspire Navigator Opportunities**Note 1****Part 1, *Live Presenter***

Page 1.4 – Have a student demonstrate how to change n , LMR, and a , and b . This requires skill with grabbing and moving.

Note 2**Part 2, *Quick Poll***

Use the Question pages, page 1.5-8 as ready-to-use *Quick Polls*.

Note 3**Extension 1, *Send to Class***

Either with the area.tns file open (available with this activity on TI's [Activities](#)) or from the Class workspace, use *Send to Class* and change the handheld destination folder to **MyLib**.

Note 4**Extension 1, *Live Presenter***

Have at least one student demonstrate the process of putting the file into the MyLib or Refreshing the Library.

Note 5**Extension 2, *Screen Capture***

Do a whole class screen capture. Press ctrl+0, or change Zoom Percentage under View to “Fit All.” With the students on page 3.2, and using the function $f_1(x) = -x^2 + 3x$, have the students pick an approximation method that will overestimate the area. Some students should pick right and others will pick midpoint. Discuss why they made the choice they did. Compare both of these answers. Use the + near the bottom center to add more subintervals to confirm.