Exponential Reflections

About the Lesson

In this activity, students will determine that the inverse of the exponential function is the natural log function by plotting the inverse of exponential solution points. As a result, students will:

- Analyze the function $y = e^x$, its corresponding inverse function $y = \ln x$, and their reflection about the line $y = x$.
- Analyze the function $y = 10^x$, its corresponding inverse function $y = \log(x)$, and their reflection about the line $y = x$.

Vocabulary

- line reflections
- natural logarithmic function
- exponential function
- inverse functions

Teacher Preparation and Notes

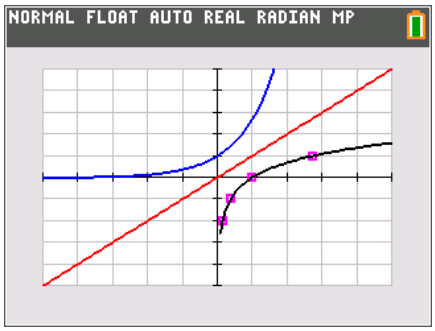
- Students should be somewhat familiar with the concept of basic logarithms in order to complete this activity.
- This activity is suitable for an Algebra 2 class or as a refresher activity for a Precalculus class.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

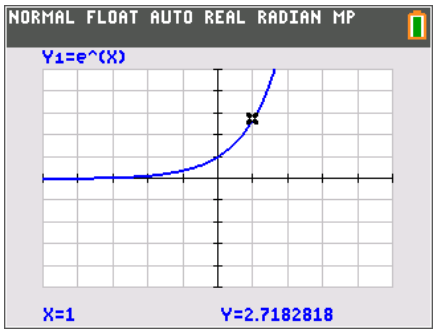
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Exponential_Reflections_Student.pdf
- Exponential_Reflections_Student.doc

Problem 1 – Reflecting the Exponential Function

Students begin by graphing the function, $y = e^x$ and recording the *y*-values for certain *x*-values by using the **Table**.



The students then find the inverse of these collected points by switching the *x*- and *y*-values and plotting them.

1. What would the inverse of this graph look like?

Answer: The inverse graph would be increasing, concave down, passing through (1, 0), having a domain of $(0,\infty)$ and a range of $(-\infty\infty)$.

Tech Tip: If your students are using the TI-84 Plus CE have them turn on the GridLine by pressing 2nd zoom [format] to change the graph settings. If your students are using TI-84 Plus, they could use GridDot.

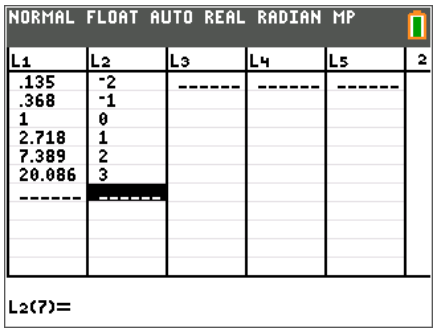
2. Record the *y*-values under the original *y*-value column in the table below.

Next record the inverses of each point by switching the *x*- and *y*-values and recording the results in the inverse columns in the table below.

Answers:

Original *x*-value	Original *y*-value	Inverse *x*-value	Inverse *y*-value
–2	0.135	0.135	–2
–1	0.368	0.368	–1
0	1	1	0
1	2.718	2.718	1
2	7.389	7.389	2
3	20.086	20.086	3

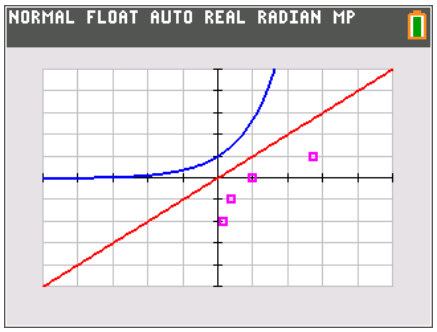
The students then find the inverse of these collected points by switching the *x*- and *y*-values and plotting their scatter plot.



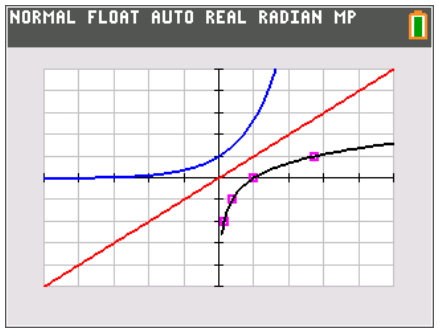
3. What do you notice about the plotted values?

Answer: They appear to reflections of the points on the graph of $y = e^x$ about the line $y = x$.

As the students look at the graph and look for a pattern, it may be helpful for them to plot the function $y = x$ (in **Y2**) to help them see the reflection.



After noticing the reflection of the original function, the students are asked to solve the inverse function by hand.



4. Find the inverse of $y = e^x$. This is done by switching x and y (exchanging the input with the output) in the equation and solving for y.

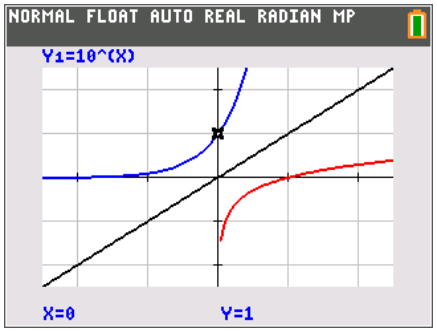
Answer:

$$y = e^x$$
$$x = e^y$$
$$\ln(x) = \ln(e^y)$$
$$\ln(x) = y \ln e$$
$$\ln(x) = y$$

Have students graph their resulting function to show that it matches the scatter plot.

Extension – Reflecting $y = 10^x$

In this part of the activity, students repeat the process from Problem 1 using $y = 10^x$. Students may need to "zoom in" to see portion of the graph of $y = \log(x)$ that exists close to the y-axis. Pressing zoom, selecting **2:Zoom In** and pressing enter will zoom in at the origin and allow the graph to be viewed a little more appropriately.



5. Find the inverse of $y = 10^x$.

Answer:

$$y = 10^x$$
$$x = 10^y$$
$$\log(x) = \log(10^y)$$
$$\log(x) = y \log 10$$
$$\log(x) = y$$

The students will determine that the inverse of $y = 10^x$ is $y = \log(x)$.