NUMB3RS Activity: Spies Like Us
Episode: The Art of Reckoning

Topic: Probability Game **Grade Level:** 9 - 12
Objective: To analyze the probability of winning a game partially based on chance
Materials: Three different colored dice, or a TI-83 Plus/TI-84 Plus graphing calculator with the Probability Simulation App installed
Time: 20 - 30 minutes

Introduction

In "The Art of Reckoning," Charlie discusses the security of a prison. "It's a hyper-secure system, but prisoners have nothing to do except think about how to crack it. Like two opposing armies…" As he talks, a fort is shown under siege with a spy trying to break in, using ladders, underground tunnels, and catapults. Charlie explains that analyzing such a situation involves determining the probability that enemy lines are breached by considering the ratio of vulnerable points to the resources required to defend that point. In this activity, students analyze a game in which a spy is attempting to break into a fort.

Discuss with Students

In the version of the game in the main activity, Player A moves the spy, and Player B moves the guards. Since the movement of the guards is entirely dependent on the roll of dice, Player B has no control, and this version of the game is virtually a one-player game. As a result, this game can be analyzed using probability, and students can analyze various situations during the main activity.

To familiarize students with the rules, you may want to play a game with them as a class. Project the game board on the overhead projector, and have students control the spy as you control the guards. Do not discuss strategy during this game, but be sure to emphasize any of the "tricky" rules of the game, such as when two guards try to move to the same unoccupied entrance (in which case, neither of them move).

The activity involves rolling three dice. This can be done with actual dice or the Roll Dice option on the Probability Simulation App on the TI-83 Plus/TI-84 Plus graphing calculator. When using the Probability Simulation App for this activity, choose **SET** to change the number of dice to 3. Choose **TABL** to display the sequence of rolls. The Probability Simulation App can be downloaded from the Web site below:
http://education.ti.com/educationportal/sites/US/productDetail/us_prob_sim_83_84.html

After playing the game, you may wish to discuss the distribution of the guards with the class. In essence, each guard's location at the end of a game resembles a normal distribution, but the distribution is skewed because of the width of the board. When a guard reaches either end, it is not possible for him to move any farther in that direction, so the last entrance becomes more likely.

A variation of the game appears in the Extensions. Although more difficult to analyze, this version of the game gives more control to Player B, so is more interesting to play.

Student Page Answers:
1. *5/9; there is a 2/3 × 2/3 = 4/9 chance that neither guard will catch the spy, so there is a 1 – 4/9 = 5/9 chance that at least one of them will catch him. Note that on this move, both the green and blue guards are allowed to move to 4.* **2.** *The optimal strategy for the spy is to move into the lower right corner on his next turn. To be caught, the blue guard will have to move to the right twice, and the probability of that happening is 1/3 × 1/3 = 1/9.* **3.** ${}_7C_3 = 35$ *ways.* **4.** Answers will vary, but one reasonable strategy is to try to move to one of the entrances at an end of the wall. On average, the guards are less likely to end up there.

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Name: _____ Date: _____

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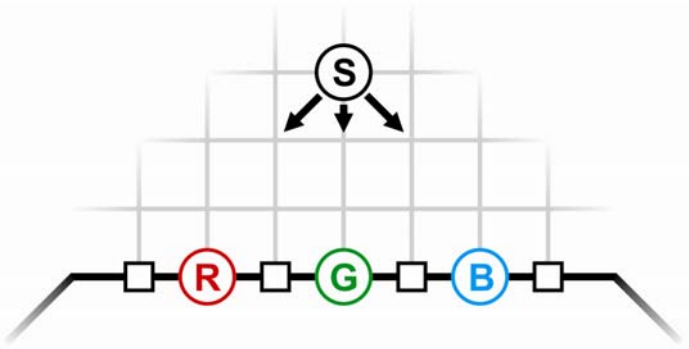
The wall of the fort has seven entrances guarded by three guards. Each guard is positioned at an entrance. If the entrances are numbered consecutively 0-6, then the red guard (R) starts at entrance 1, the green guard (G) at 3, and the blue guard (B) at 5, as shown below. The spy (S) starts at the center of the top row.



The game proceeds as follows:

- On each turn, Player A (the spy) moves forward one square on the grid, either diagonally or straight ahead.
- After the spy moves, Player B (the guards) moves as follows:
  - Player B rolls three dice, one for each guard. (It is helpful to use three different colored dice, one for each guard.) Each guard moves according to the number rolled:
    - 1-2 – move one entrance to the left
    - 3-4 – remain at the same entrance
    - 5-6 – move one entrance to the right
  - Guards may only move to an unoccupied entrance. (Note that the red guard cannot move left past position 0 and the blue guard cannot move right past position 6 – either one could get "stuck.")
  - If the dice indicate that two guards must move to the same unoccupied entrance or switch entrances, then *neither* guard moves.
- Players A and B alternate for two turns.
- On his third turn, the spy will reach the wall (indicated by the heavy black line). On this turn, he *must* move to an unoccupied entrance. If this is not possible, then he is caught. If this is possible, then the guards still get one more turn. If the dice dictate that at least one of the guards moves to the entrance where the spy is, then he is caught. (On this last turn, two guards *are* allowed to move to the same entrance.) Otherwise, the spy enters the fort unnoticed.

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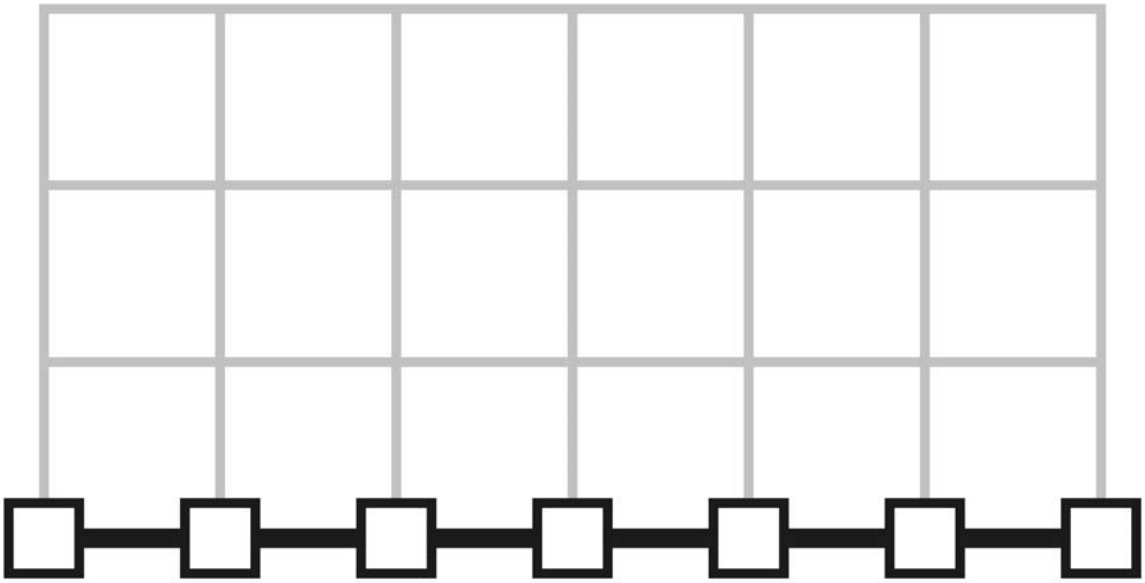
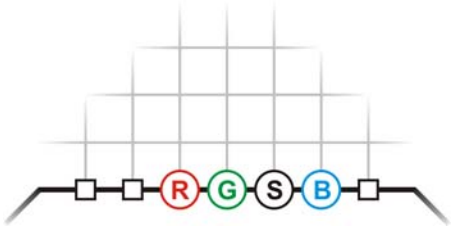
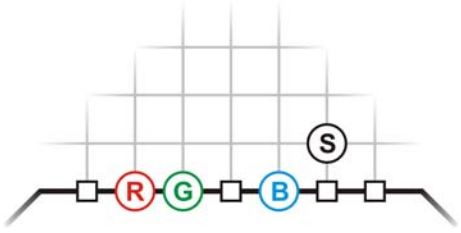
Play this game several times with a partner using the game board below. Use coins or other markers as the spy and the guards. After several games, swap roles as Player A and Player B.



1. Suppose that after three turns in a game, the guards are at entrances 2, 3, and 5, and that on his third turn, the spy had no choice but to move to the entrance shown below. What is the probability that he will be caught on the guards’ final turn?



2. In the game shown below, the spy has taken two turns, but the guards have taken only one. It is the guards’ turn to move. If the spy moves with his best strategy, where should he move, and what is the probability that he will be caught?



3. In how many different ways can the guards be arranged at the entrances after three turns, assuming that no two guards finish at the same entrance?

4. What is a good strategy for the spy to use for this game?

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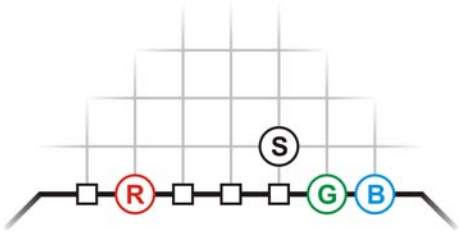
The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Variation

In the game described above, Player B moved the guards based on rolls of the dice. In this variation, Player B has more control over the movement of the guards. All of the previous rules apply, except as noted below:

- If the dice indicate that two guards must move to the same unoccupied entrance, then Player B can choose to move one of them to that entrance.
- If triples (all three dice show the same number) are rolled, Player B may form a blockade—the green guard is placed at the entrance directly in front of the spy, and the red and blue guards are placed adjacent to him.
- If doubles (two dice show the same number) are rolled, Player B can either take the results of the roll, or he can choose instead to move just one of those guards any number of spaces left or right, without passing an adjacent guard. For instance, if the guards are positioned as shown below and double 4s are rolled for the red and green guards, both the red and green guards can be moved one entrance to the left; or, Player B and choose instead to move the red guard one, two or three entrances to the right or to move the green guard one, two, or three entrances to the left.



Play this variation with a partner. Would you rather be Player A or Player B?

Additional Resources

John Horton Conway is one of the masters of mathematical games. The following is a good book for math games similar to the one in this activity.

Conway, J. H. *On Numbers and Games*. 2nd ed. London: A. K. Peters, Ltd., 2000.

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