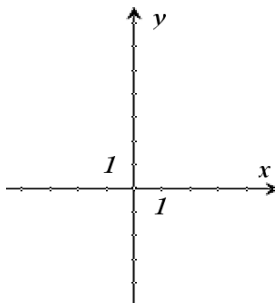


**Part 1 – Warm Up**

1. When is  $y = e^x$  increasing and when is it decreasing? Sketch the graph.



Set your TI-89 Titanium to Split Screen mode. Press **[MODE]**. Change **Split Screen** to **LEFT-RIGHT**, **Split 1 App** to **Y= Editor**, and **Split 2 App** to **Table**. Then press **[ENTER]** to save changes.

In **y1**, enter  $x^n$ . In **y2**, enter  $e^x$ . Press **[HOME]** and store 2 as the value for  $n$ . To update the values in the table, press **[2nd] + [APPS]**. Examine the values for **y1** and **y2**. Then press **[2nd] + [APPS]** and enter new values for  $n$ , updating the table for each new value.

2. For what values of  $x$  greater than 2 is  $y = e^x$  is greater than  $y = x^3$ ? Sketch  $y = x^3$  on the same graph.

3. Fill in the blank:  $e^x$  is greater than  $x^4$  when  $x > \underline{\hspace{2cm}}$ .

As you have seen, raising a number to an increasing power, exponential growth, is more dramatic growth than raising an increasing number to a power.

Recall the Power of a Power Exponent rule:  $b^{a \cdot x} = (b^a)^x$ .

4. If  $y = e^{ax}$ , exponential growth occurs for what values of  $a$ ? If  $y = b^x$ , exponential growth occurs for what values of  $b$ ?
5. If  $y = e^{ax}$ , exponential decay occurs for what values of  $a$ ? If  $y = b^x$ , exponential decay occurs for what values of  $b$ ?
6. Let  $y(x) = 3e^{kx}$ . Solve for  $k$  when  $y(4) = 9$ . Show your work.

**Part 2 – Infestation (Exponential Growth)**

The rate of increase of bugs is proportional to the number of bugs in a certain area. When  $t = 0$ , there are 2 bugs and they are increasing at a rate of 3 bugs/day. (This first sentence can be written as a differential equation, with  $k$  the constant of proportionality. The other info is called the *initial condition*, or IC.)

1. If  $b$  is the number of bugs, express the first sentence as a differential equation.
2. The first step to solve this DE is to separate the variables. Do this step.
3. Next, integrate both sides of the equation.
4. Why do you not need to put a “+  $C$ ” on both sides of the equation after integrating?

To write a particular equation for the number of bugs as a function of time, apply the initial condition and solve for  $b$ .

5. Recall, when  $t = 0$ , there are 2 bugs and they are increasing at a rate of 3 bugs/day. So What is  $k$ ?
6. Find the particular equation for the number of bugs as a function of time by applying the initial conditions. Show work to solve for  $b$ .

Graph  $b(t)$  in **y1(x)** to check your answers to the following questions. Be sure to show all work.

7. How many bugs are there on the beginning of the 2nd day? At the beginning of day 3?
8. When will there be approximately 1,200 bugs? Show the equation and what you substituted.

**Part 3 – Extermination (Exponential Decay)**

1. Mr. Exterminator arrives to save the day when there are 1,600 bugs. He applies some poison which produces an exponential decay such that the rate of decrease is proportional to the number of bugs. At the start of day 2 of bug elimination,  $t = 2$ , you estimate 600 bugs. Find the particular solution for the number of not-dead-yet bugs as a function of time,  $d(t)$ . Show all of the steps to solving this separable differential equation.

Graph the function  $d(x)$  in **y1** to answer the following questions.

2. Mr. Exterminator says the poison lasts for 10 days. Use the **Trace** tool (press **F3** while on the Graph screen) to determine how many bugs are still alive when  $t = 10$ .

$$d(10) = \underline{\hspace{2cm}}$$

3. Should you have him come back for a second application? Explain.
4. About how many days until there is only one bug left? Show work or explain your method.

**Extension (CAS deSolve)**

From the “Infestation Problem,” the rate of change of  $b$  was directly related to  $b$ . This can be written as  $b' = k \cdot b$ , where  $b$  is a function of time.

The syntax for the deSolve command (**F3: Calc > C:deSolve()**) is:

**deSolve(diff\_eq,indep\_var,depend\_var)**, e.g.  $\text{deSolve}(b'=k*b,t,b)$ .

1. Enter the command **deSolve(b'=k\*b,t,b)** on the HOME screen. What is the result?
2. We know  $k = 1.5$  and the initial condition (IC)  $b(0) = 2$ . What value of @1 (the constant) satisfies the IC? Graph your equation to verify.

## Infestation to Extermination

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On the HOME screen, enter the **deSolve** command again, but this time enter the initial condition  $b(0) = 2$ : **deSolve(b'=1.5b and b(0)=2,t,b)**. This will return the particular solution.

3. What is the particular solution?

This answer should confirm the algebraic solution from Part 2 and the graph.

NOTE: It is important to be able to show your work—separate the variables and integrate, being careful of the constant of integration. However, for the next problems, use **deSolve** to explore other differential equations.

Use  $y'$  for  $\frac{dy}{dx}$ .

4. What is the general solution for  $\frac{dy}{dx} = -\frac{x}{y}$ ?

5. What is the graph of the solution to this separable DE?

6. In the previous examples, the constants were labeled as @1, @2, @3, etc. What does this mean?

7. Use CAS to find the general solution for:

a.  $y' = \frac{x}{y}$

b.  $x \cdot y' - y = 0$

c.  $y' = 2x$

d.  $y' - 6x^2 = 0$