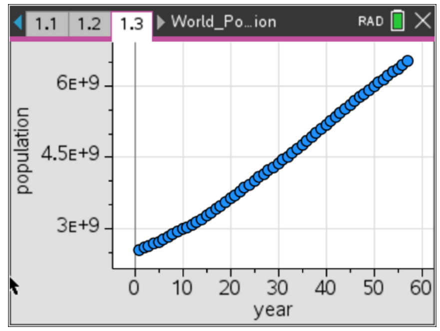
World Population

Name ______________________________

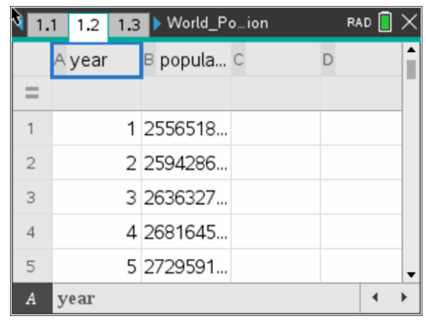
Class ______________________________



Population growth can be modeled using various exponential functions. In this activity, students will use two different methods to find the best model for given data. The data will represent the midyear world population from the years 1950 – 2023, where year 1 represents the year 1950.

The data will be downloaded from the file ***World_Population.tns***.

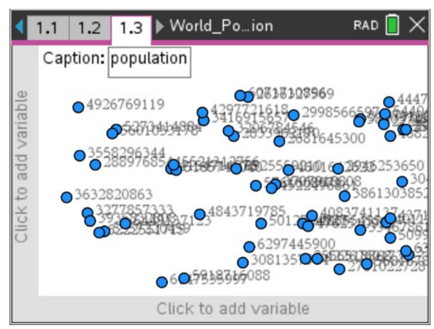
Problem 1 – Find an exponential equation by hand using two points.



In this part, you will find an exponential equation using two points. Begin observing the data.

Move to page **1.2**. You will now see the world population data in columns A and B.

1. What kind of function would best model the data?



To examine the data, you will make a scatter plot.

Move to page **1.3**. Add the horizontal (year) and vertical (population) variables to fit the data to the screen. See the picture to the right.

2. Now find an exponential model to fit the data by using two points ($x1$, $y1$) and ($x2$, $y2$). To do this on the scatter plot, press **menu**, **4 Analyze**, **A Graph Trace** and use the left and right arrows to see the coordinate values.
   (a) Pick two points that are spread throughout the data and write the values in the space provided:

   Coordinate 1: $x1$ = ________ $y1$ = __________________________

   Coordinate 2: $x2$ = ________ $y2$ = __________________________

The general formula for an exponential function is $\boldsymbol{y = a(b^x)}$ where *a* is the initial value and *b* is the multiplier or in this case the growth rate.

(b) Find the equation through the two points by substituting the values into the general exponential formulas:

$y1 = a(b^{x1})$ ________ = ____________

$y2 = a(b^{x2})$ ________ = ____________

Then divide the equations and solve for *b*.

$$\frac{y2}{y1} = \frac{a\left(b^{x2}\right)}{a\left(b^{x1}\right)}$$ ________________________

(c) What happens to the value of *a* when the two equations are divided?

(d) What is your value of *b*?

Move to page **1.4** and store this value in for the letter ***b***. Type in the value for *b* and press **ctrl**, **var (sto)**, **enter**.

(e) According to your *b* value, by what percent is the world population growing?

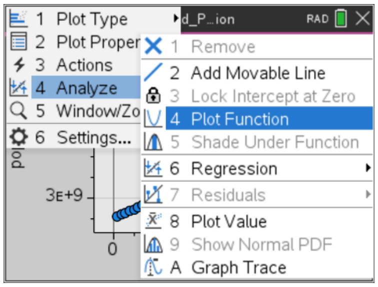
Now substitute the value of *b* back into either of the equations above to find the value of *a*. Follow the steps above to store this value as ***a***.

(f) What is your value of *a*?

(g) What is your exponential equation?

(h) Look at your equation, what is your initial population?

To graph your equation with the data, go back to the scatter plot on page **1.3**, press **menu**, **4 Analyze**, **4 Plot Function**. Type in the function $a \cdot b^x$ and press **enter**.



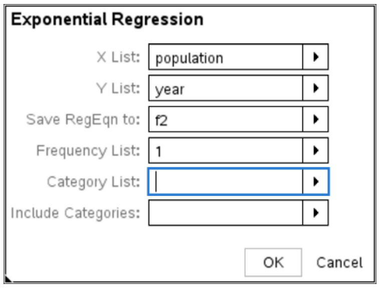
(i) How well does your equation model the data? Explain.

(j) What will the population be in the year 2030?

(k) What was the population in the year 1890?

Problem 2 – Finding the exponential regression.

Now you will find an exponential regression equation to fit the data.



3. On page **1.4**, press **menu**, **6 Statistics**, **1 Stat Calculations**, **A Exponential Regression**. Make sure the X and Y lists are year and population, respectively, the equation is stored in $f2(x)$, and then press **enter**.
   (a) Discuss with a classmate which model seems to fit the data more accurately. Summarize your discussion here.

   (b) What is the exponential regression equation?

   (c) What is the initial population?

   (d) By what percent is the world population growing?

   (e) How do these two values compare with the ones in Problem 1?

(f) How well does the exponential regression equation model the data?

(g) What will the population be in the year 2030?

(h) What was the population in the year 1890?

(i) Which model do you feel is best? Why?

(j) How would you find out what year the world's population will reach 8.9 billion people?

(k) Graphically use the exponential regression equation to find the year.

Problem 3 – Extension beyond population

A botanist has been studying the rare plant called the Lady's Slipper Orchid that was at first thought to be extinct but was once again discovered in the 1930s. She is trying to foster its growth. In 2020 (t = 3), she has 36 plants, and in 2024, she has 108 plants.

The number of plants being cultivated and studied can be modeled by the function $L(t) = a \cdot b^t$, where $L(t)$ is the number of plants during year t, and t is the number of years after 2017.

4. (a) Use the given data to write two equations that can be used to find a and b.

   (b) Find the values of a and b as decimal approximations.

   (c) Use your function, $L(t)$, to approximate the number of plants the botanist will have in 2028.

   (d) Using your function, $L(t)$, when will the number of plants reach 200 plants.