Math Objectives

- Students will relate inequalities in one variable (on a number line) to inequalities in two variables (on the coordinate plane),
- Students will also identify possible solutions to inequalities in one and two variables, and recognize the pattern in the coordinates of the solutions,
- Students will graph linear inequalities with vertical and horizontal boundary lines.

Vocabulary

- boundary lines
- coordinate plane

About the Lesson

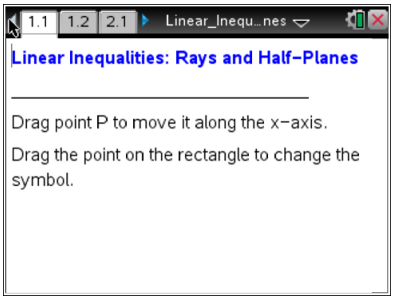
- This lesson introduces linear inequalities with vertical and horizontal boundary lines by relating them to simple inequalities on the number line.
- Inclusive and non-inclusive inequalities are explored by relating dotted and solid boundary lines in the coordinate plane to open and closed circles on the number line.
- Students will drag a point on the boundary line and observe changes in the corresponding inequalities in one and two variables and the coordinates of the points in the solution set. They will then change the inequality symbol in the coordinate plane and observe the effects on the corresponding inequality on the number line.
- This activity can lead into graphing linear inequalities with boundary lines. (See Linear Inequalities in Two Variables)

TI-Nspire™ Navigator™ System

- Send out the *Linear_Inequalities_Rays_and_Half-Planes.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Materials:

Student Activity

- Linear_Inequalities_Rays_and_Half-Planes_Student.pdf
- Linear_Inequalities_Rays_and_Half-Planes_Student.doc

TI-Nspire document

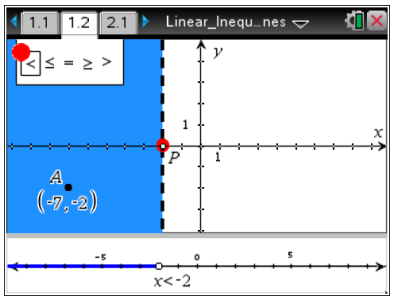
- Linear_Inequalities_Rays_and_Half-Planes.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, make sure they have not selected multiple objects. Press esc to release points. Check to make sure that they have moved the cursor (arrow) until it becomes a hand () getting ready to grab the point. Also, be sure that the word point appears. Then select ctrl to grab the point and close the hand (). They can change the inequality symbol selected by dragging the upper left vertex of the rectangle.

Move to page 1.2.

Note**:** Page 1.2 of the .tns file begins with $x < -2$.



1. Move point *P*. Describe the changes that occur on both the number line and the coordinate plane as you move the point. What stays the same?

   Answer: As you move point *P,* the dotted line in the coordinate plane moves, and the point on the number line moves. The value on the right side of the inequality at the bottom of the screen changes. The area of the shaded region increases or decreases and the darker portion of the number line increases or decreases. The coordinates of point *A* stay the same.

   Teacher Tip: Students may not notice the number line graph at the bottom of the screen and begin to see the relationship between the one and two-dimensional depictions of the inequality.

2. a. Move point *P* so that the inequality at the bottom of the screen is $x < -2$. Move point *A* around in the shaded area. How is the *x*-coordinate of point *A* related to the number line and inequality statement at the bottom of the screen?

   Answer: The *x*-value for all of the ordered pairs in the shaded region is less than –2. These are the numbers graphed on the number line that satisfy the inequality statement $x < -2$ at the bottom of the screen.
   When the *x*-coordinate of point *A* is less than –2, then point *A* is "above" the darkened ray at the bottom of the screen. When the *x*-coordinate of point *A* is –2, point *A* is directly about –2 on the number line. Otherwise, the *x*-coordinate of point

A is above the non-darkened part of the number line.

b. Is it possible to move point *A* to find an ordered pair in the unshaded region where the *x*-coordinate in the ordered pair is less than –2? Why or why not?

Answer: No, it is not possible to find an ordered pair in the unshaded region for which the *x*-coordinate is less than –2. When point *A* is in the unshaded region, it is to the right of the line $x = -2$, thus all *x*-values will be greater than –2.

3. a. How does the solution set for $x < -2$ on the number line differ from the solution set in the coordinate plane?

Answer: The solution set on the number line would be all individual numbers less than –2, so numbers such as –2.5, –4, –10, and so on would be in the set. In the plane, the solution set would be all ordered pairs where the *x*-coordinate was less than –2, e.g., (–3, 4), (–4.5, 4).

b. Which of the following ordered pairs are in the solution set for $x < -2$ in the coordinate plane?

(–10, 3) (–2, 2) (0, –4) (–1.5, 0) (–2.5, –2)

Answer: (–10, 3) and (–2.5, –2)

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

4. Grab the point on the box surrounding the inequality symbol. Move the box to change the inequality symbol selected. Describe the changes that occur on both the number line and the coordinate plane. What stays the same?

Answer: When the “equals” sign is selected, the line in the coordinate plane is solid, the circle on the number line is closed, and there is no shading. As you change the inequality symbol, the vertical line will be dotted, except for the "≤ or ≥,” sign, in which case it is solid. Similarly, the point on the number line will be open, except for the "≤ or ≥" sign, in which case it is closed or filled in. The shading on the plane and the darkening of the number line will change from the left to right. The locations of point *P*, the vertical line, and the circle on the number line do not change.

5. Describe the characteristics of the graphs of each of the following:

For the Given Inequality	$x < 3$	$x \leq 3$	$x = 3$	$x \geq 3$	$x > 3$
In one variable, is the circle open or closed?	**Open**	**Closed**	**Closed**	**Closed**	**Open**
In two variables, is the boundary line dotted or solid?	**Dotted**	**Solid**	**Solid**	**Solid**	**Dotted**
Is the shading to the right or left?	**Left**	**Left**	**Neither**	**Right**	**Right**

6. Give an example of an inequality such that the graph has:
   a. an open circle and dotted line
      Answer: Examples can be either less than or greater than statements but should not have an equals sign.

   b. a solid line and shading to left
      Answer: Examples could be of the form $x \leq a$, $a \geq x$, or even $x \geq a$ for any number a, although at this stage the latter is unlikely to be suggested.

   c. shading to the right
      Answer: Examples could have $x > a$ or $x \geq a$ for some number a as the relationship as well as examples such as $a < x$, $a \leq x$, $-x < a$, or $-x \leq a$.

   d. shading to the left and a closed circle
      Answer: Examples could use $x \leq a$ as the relationship as well as $a \geq x$ or $x \geq a$ for any number a.

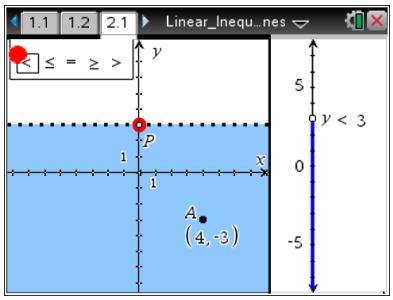
> **Teacher Tip**: Note that for parts b, c, and d students will most likely respond with examples of the form $x < a$ but by the end of the lesson should realize that there are other possible ways to state a relationship that will have shading to the left. This is addressed in question 7 below, and the role of the negative sign is addressed in the activity Order and Inequalities: Multiplication by Negative Numbers.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

7. Tejal says, "The graph of $x < -12$ in the coordinate plane would have a dotted boundary line and be shaded to the right." Do you agree? Why or why not?

   Answer: Tejal is partially correct. The graph of $x < -12$ does have a dotted boundary line, but it is shaded to the left.

Move to page 2.1.
Note**:** Page 2.1 of the .tns file begins with $y < 3$.



8. Observe the coordinates of point A as you move through the shaded region, to the line, and through the unshaded region.

   a. Use what you observe to explain why the inequality $y < 3$ describes the solution set for the shaded area on the coordinate plane.
   Answer: The inequality $y < 3$ will describe all of the ordered pairs in the shaded region of the plane. Every ordered pair in that region will have a y-coordinate less than 3 although the x-coordinate can vary and is restricted only by the window on the screen.

   b. How is the solution set for $y < 3$ for the plane different from the solution set for $y < 3$ on the number line?
   Answer: The inequality $y < 3$ will describe both the values in the solution set graphed on the number line and all of the ordered pairs in the shaded region on the bottom half of the plane.

> **Teacher Tip:** Ask students how they would know whether to graph $y < 4$ in a plane or on a number line. Point out that they need more information in order to decide, such as whether the graph was to be in one or two dimensions.

9. Compare the graphs of $x < 4$ and $y < 4$ as inequalities graphed in the coordinate plane.
   a. How are they similar?
   Answer: Both graphs contain dotted boundary lines.

b. How are they different?

Answer: The graph of $x < 4$ contains a vertical boundary line whose x-intercept is the point (4, 0). The graph of $y < 4$ contains a horizontal boundary line whose y-intercept is the point (0, 4). The graph of $x < 4$ is shaded to the left of the boundary line, while the graph of $y < 4$ is shaded under the boundary line.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The possible solutions to inequalities in one and two variables.
  - With inequalities in one variable (on the number line), possible solutions exist as single values for the given variable.
  - With inequalities in two variables (on the coordinate plane), possible solutions exist as ordered pairs.
- That the solution set for an inequality in the coordinate plane with only one variable is a half plane with a horizontal or vertical boundary line.
- How to graph inclusive and non-inclusive linear inequalities.
  - Non-inclusive inequalities have an open circle on the number line or a dashed boundary line in the coordinate plane.
  - Inclusive inequalities have a closed circle on the number line or a solid boundary line in the coordinate plane.

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Note 1

Question 3, *Quick Poll*: Use the Open Response feature and have students enter the ordered pair(s) that answer question 3b.

Note 2

Question 6, *Class Capture*: For each question in number 6 (a, b, c, d), take a class capture of each student and have the students decide which are correct and which are not, and why.