Area and Perimeter of Regular Polygons -- An Extension

Activity 1

The amount of floor space in a home is important to the comfort of its owners. Many colonial houses had rectangular floors. In this case, the area of the floor is relatively easy to determine. The floor of a large tipi, however, could be a regular polygon with as many as 18 sides.

Exploration

How could you find the area of a floor shaped like an 18-sided regular polygon? The area of a geometric figure refers to the region enclosed by it. The area of a polygon may be found by estimating, by using a formula, or by other means.

In this exploration, you calculate area by inscribing a regular polygon inside a circle. An **inscribed polygon** is one in which each vertex lies on a circle. The **radius** of an inscribed regular polygon is equal to the radius of the circle.

For example, Figure **1** shows a square inscribed in a circle, with the radius of the square drawn from the square's center to one of the vertices.

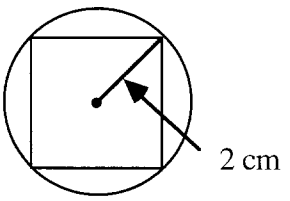

Figure 1: Inscribed square with a radius of 2 cm

a.
1. Construct a circle on a geometry utility. Record its radius, area, and perimeter.
2. Select three points on the circle to represent the vertices of an inscribed triangle.
3. Connect the center of the circle to each of the three points to form three central angles. A **central angle** is an angle with its vertex at the center of a circle.
4. Move the points along the circle, one at a time, until the central angles all have equal measures.
5. Connect the three points on the circle to form a regular triangle.

b.
1. Use the geometry utility to calculate the triangle's area and perimeter.
2. Record this information in a table.

c. Using a process similar to that described in Part **a**, inscribe a regular quadrilateral, pentagon, hexagon, octagon, and 18-sided polygon in circles of the same radius.

d. In your table from Part **b**, record the area and perimeter of each regular polygon, as well as the area and perimeter of the circle in which they were inscribed. **Note:** Save your data for use in the discussion.

Discussion

a. How did you determine the locations of the vertices for your inscribed polygons?

b. In Parts **a** and **c** of the exploration, why does creating congruent central angles guarantee that a regular polygon will be formed?

c.
1. What methods do you know for determining the area of regular polygons?
2. What are the advantages and disadvantages of each one?

d. Consider a set of regular polygons inscribed in the same circle. As the number of sides in a polygon increases, describe what happens to each of the following characteristics:

1. the shape of the polygon
2. the area of the polygon
3. the perimeter of the polygon.

e. A **tangent** to a circle is a line, segment, or ray that intersects a circle in one point and is perpendicular to a radius at that point. Consider a pentagon whose sides are each tangent to a circle, as shown in Figure **2** below. In this case, the polygon **circumscribes** the circle.

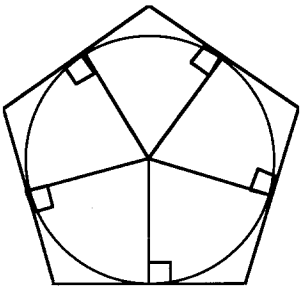

Figure 2: A circle circumscribed by a regular pentagon

Consider a set of regular polygons that circumscribe the same circle. As the number of sides in a polygon increases, describe what happens to each of the following characteristics:

1. the shape of the polygon
2. the area of the polygon
3. the perimeter of the polygon.

f. Figure **3** below shows a regular pentagon inscribed in a circle. In this case, $\overline{AG}$ is an **apothem**, a segment whose measure is the perpendicular distance from the center of a regular polygon to one of its sides.

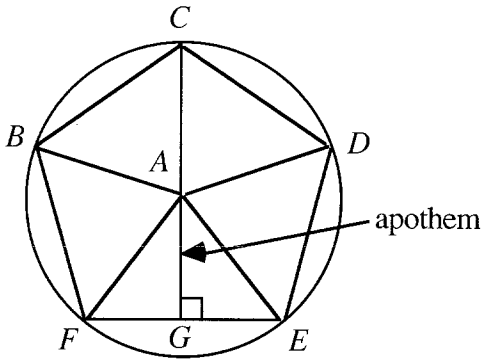

Figure 3: A regular pentagon

One formula for the area of a regular polygon is

$$A = \frac{1}{2}ap$$

where a is the length of the apothem and p is the perimeter of the polygon. (See the Level 1 module, "A New Look at Boxing.")

1. As the number of sides of an inscribed regular polygon increases, what do the values of a and p approach in relation to the circle?
2. How does this affect the formula for the area of a regular polygon given above?