

Probable Factors

Student Activity

7 8 9 10 11 12



Introduction

What is the probability that a randomly generated quadratic can be factorised? This investigation looks at a significantly reduced set of quadratics using dice to generate the coefficients. The coefficient of x^2 is 1. The remaining coefficients are generated randomly using dice. Two dice are rolled; the numbers appearing uppermost are added together and form the x coefficient (b) in the quadratic: $y = x^2 + bx + c$. Another two dice are rolled; the numbers appearing uppermost multiply together and form the constant (c). What is the probability this quadratic can be factorised?

- Over the rational number field?
- Over the real number field?

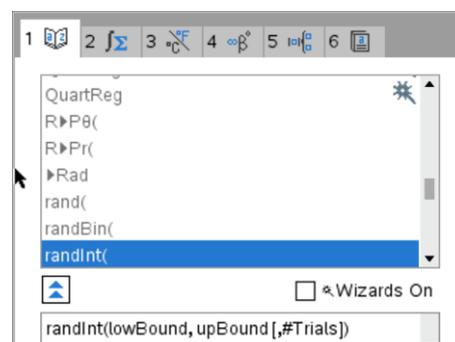
Introduction – Setting up the simulation

Start a new document and insert a spreadsheet. The spreadsheet will be used to generate the random integers (whole numbers) in place of a die.

In cell A1 type the formula: `=randInt(1,6)`

This produces a random integer between 1 and 6.

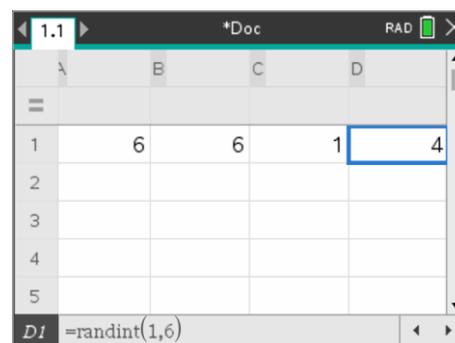
Note: In the spreadsheet application the fastest way to access the **randInt()** command is through the catalogue.



The formula in cell A1 can be copied into cells B1, C1 and D1.

Click on cell A1, press **CTRL + C** to copy, move to cell B1 and press **CTRL + V** to paste.

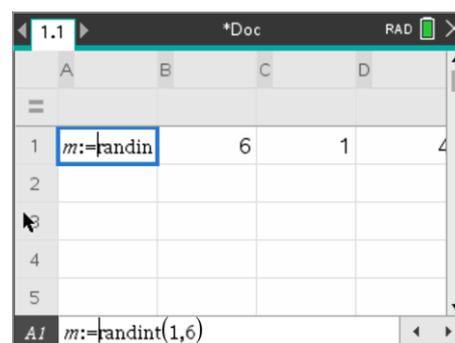
Repeat this process for cells C1 and D1.



The simulated dice rolls need to be stored. Select cell A1 and press **CTRL + VAR** and store the roll in 'm'.

Repeat this for cell B1, store as 'n'; store cell C1 as 'p' and cell D1 as 'q'.

Note: The simulated dice rolls become bold once they have been stored. Press **CTRL + R** to simulate dice rolls and observe how each cell contains a new outcome each time CTRL + R is pressed.

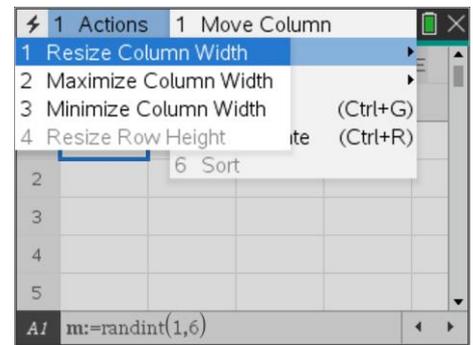




The column width can be adjusted using the Actions menu.

[Menu] > Actions > Resize > Resize Column Width

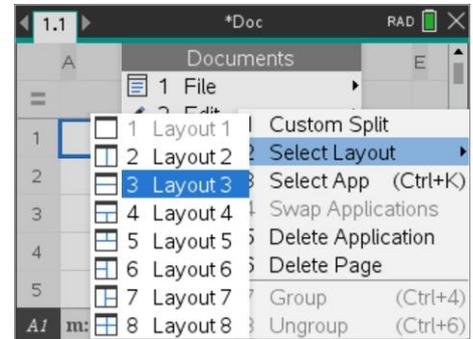
Use the left / right arrow keys to change the column width.



The next step is to split the current page into two applications.

[Doc] > Page Layout > Layout 3

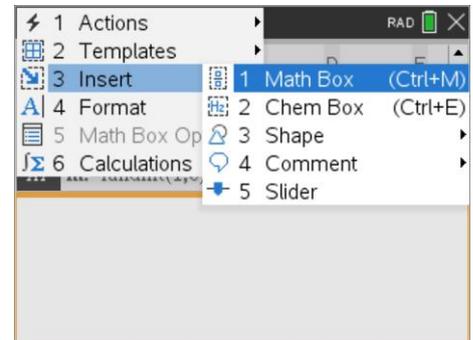
This will split the current page into two horizontal regions.



As the page is split into two you will be prompted to insert another application. Insert a **Notes** Application.



On the calculator, Ctrl + Tab performs a similar role to Alt + Tab in Microsoft Windows™, this shifts the focus between active applications.



The first step is to define a quadratic function.

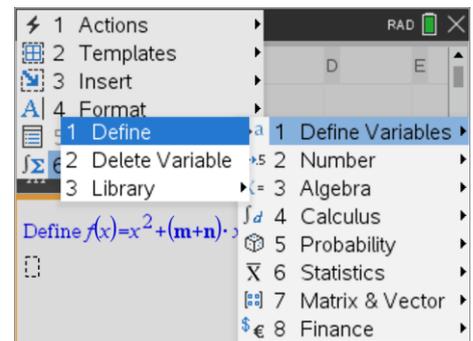
Press **Ctrl + M** to insert a Maths Box into the Notes Application.

In the Maths box define the function $f(x) = x^2 + (m+n) \cdot x + p \cdot q$

[Menu] > Calculations > Define Variables > Define

A multiplication sign must be used between the parenthesis and the x, similarly between the p and q to ensure these operations are multiplied.

Notice the defined variables m, n, p and q appear bold.

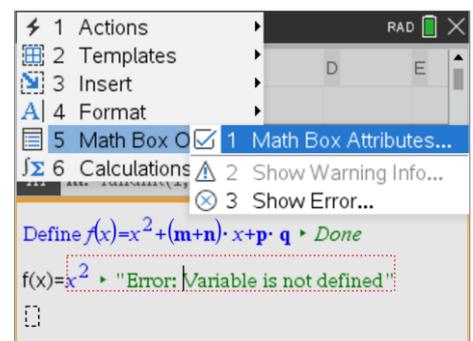


The TI-Nspire non-CAS platform does not support algebraic functionality in any form, some clever use of text and Maths boxes can be used to display the equation.

Text: f(x) =

Maths Box: x^2

When x^2 is entered into the maths box an error message will be displayed. Accept the error message and change the Maths Box Attributes through the Maths Box Options. In the input/output option, select Hide Output (error message). The result will be the expression x^2 in the maths-box. Move the cursor out of the Maths box to observe the change.



This process will be repeated for the quadratic function.

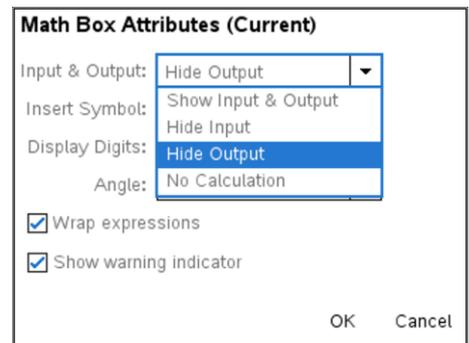
Text: +

Maths Box: $m + n$

This time, hide the input.

Maths Box: x

Hide the output (error).



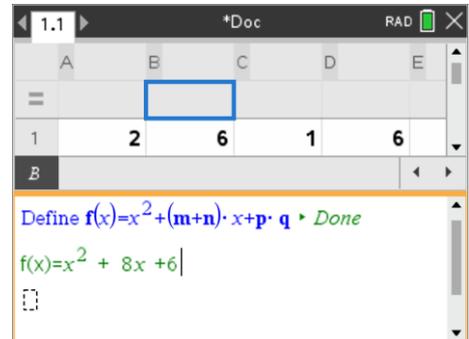
Almost done ...

Text: +

Maths Box: $p \times q$

Hide the input.

You can change the colour of the text to give the equation some consistency.

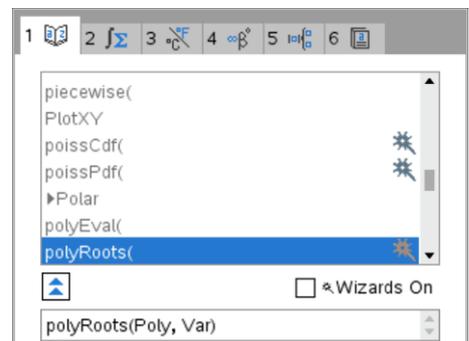


The final step in this special simulator is to find the roots.

In a maths box, press the [Catalogue] key:

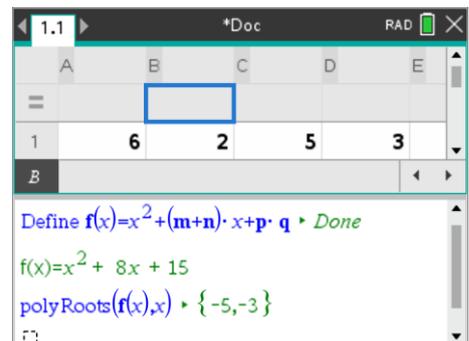
Press P and navigate to the Poly Roots command. Enter the equation definition and variable.

$$\text{PolyRoots}(f(x), x)$$



Navigate back to the spreadsheet (**Ctrl + Tab**) and press **Ctrl + R** to 'roll' the dice. Notice what happens to the quadratic equation!

In the example shown opposite: $f(x) = x^2 + 8x + 15$ has two roots since $f(x) = (x + 5)(x + 3)$ giving the roots as: $\{-5, -3\}$.



Probability through simulation

1. Press **Ctrl + R** to simulate several rolls. How do the roots of the polynomial indicate whether the quadratic factorises over:
 - a. Rational numbers
 - b. Real numbers
 - c. Doesn't factorise?
2. To get a sense of the probability of rolling a quadratic that will factorise over the rational number field, simulate 100 rolls of the dice. Watch the notes page and count how many of the quadratic equations have rational factors. Write your answer as a probability or percentage.

3. Your response to question two provides an indication of the theoretical answer, how could a better estimate of the theoretical probability be obtained?
4. Complete the lattice diagram below for the sum of the two dice; some of the answers have been completed. The lattice diagram represents all the possible coefficients for x , in the general form: $y = x^2 + bx + c$

Dice Sum	1	2	3	4	5	6
1	2					
2						
3		5				
4			7			
5						
6						

5. Complete the lattice diagram below for the product of the two dice; some of the answers have already been filled in. The lattice diagram represents all the possible constants in $y = x^2 + bx + c$

Dice Prod.	1	2	3	4	5	6
1	1					
2						
3		6				
4			12			
5						
6						

The two lattice diagrams show the individual possibilities for the coefficient (b) and the constant (c). For each coefficient there are 36 permutations making a total of $36 \times 36 = 1296$ possible equations to be factorised. Some of the equations are repeated, systematic exploration combined with some thinking and reflection will drastically reduce the time taken to explore which of these 1296 combinations factorise.

6. Suppose a [1] and a [1] are rolled on the first pair of dice to give an x coefficient of 2. The equation would be: $y = x^2 + 2x + c$ Two more dice must be rolled, the product of which form the constant (c). The lattice diagram for the product can be used to write down all the possible quadratics: $y = x^2 + 2x + 1$, $y = x^2 + 2x + 2 \dots y = x^2 + 2x + 36$
 - a. Write down all the quadratics that can be formed.
 - b. Determine which quadratics can be factorised.
 - c. Graph each of the equations indicating which ones factorised successfully over the rational number field.
 - d. Use the lattice diagram for the dice products to determine how many of the quadratics of the form: $y = x^2 + 2x + c$ will factorise.

7. Suppose a [5] and a [3] are rolled on the first pair of dice to give an x coefficient of 8.
 The equation would be: $y = x^2 + 8x + c$ Two more dice must be rolled, the product of which form the constant
 (c). The lattice diagram for the product can be used to write down all the possible quadratics: $y = x^2 + 8x + 1$,
 $y = x^2 + 8x + 2 \dots y = x^2 + 8x + 36$
- Write down all the quadratics that can be formed.
 - Determine which quadratics can be factorised.
 - Graph each of the equations indicating which ones factorised successfully over the rational number field.
 - Consider both sets of graphs and observe the value for the constant where the family of graphs no longer cross the x axis.
 - Comment on your findings.
 - Use this to predict or hypothesise the value of the constant (c) such that the family of graphs $y = x^2 + 10x + c$ will no longer cross the x axis.
 - Check your hypothesis with your teacher and discuss how this can reduce the number of possible equations to be explored in this investigation.
 - Use the “dice product” lattice diagram to determine how many of the quadratic equations with an x coefficient of 8 will factorise?
 - What other dice combinations are possible to produce an x coefficient of 8?
8. The modified lattice diagram below includes a space to record the number of quadratics that factorise for the corresponding x coefficient. Complete the lattice diagram with each of the associated results.

Dice Sum	1	2	3	4	5	6
1	2 1					
2						
3		5 2				
4			7 6			
5						
6						

9. From the completed lattice diagram determine the theoretical probability for the number of quadratics that can be factorised in this dice simulation and compare this result to the estimated probability using the simulations.

Extension

What is the probability that a quadratic produced in the same way will factorise over the real number field?