

***ADVANCED ALGEBRA***

with the  
**TI-89**

**Sample Activity: Exploration 3**

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## EXPLORATION 3

## SIMPLE PROGRAMS TO APPLY FORMULAS

THE BETTMANN ARCHIVE



*Give me a place to stand on and I will move the earth.*  
Archimedes 287-212 B.C.

When historians of mathematics try to identify the greatest mathematician of all time, they inevitably produce a list of three giants: Archimedes, Newton and Gauss. Although Archimedes lived in ancient Greece, his mathematical techniques anticipated the invention of calculus almost two millennia later! He also developed a variety of highly practical devices such as the lever

which he used to catapult huge quarter-ton rocks against an attacking Roman fleet during the siege of Syracuse in 212 B.C. Of his countless contributions to many branches of mathematics, Archimedes was most proud of his discovery that the volume of a sphere is two-thirds the volume of the cylinder in which it is inscribed. This discovery and the corresponding diagram was carved on the tomb of Archimedes.

In *Worked Example 1*, we show how to program a TI-89 calculator to eliminate the need to re-enter a formula each time we wish to perform a computation.

### WORKED EXAMPLE 1

Write a program that prompts the user to enter a value for the radius of a circle, and then displays the area of that circle.

### SOLUTION

The formula for the area,  $A$ , of a circle of radius  $r$  is given by  $A = \pi r^2$ . The keying sequence below shows how we can embed this formula in a program, **areacirc()**, that outputs the area for any input of the radius.

#### Keying Sequence

APPS 7 3 [down] [down] areacirc ENTER ENTER [down] [down]

CATALOG ClrIO ENTER ENTER F3 3 "radius" , alpha R

CATALOG approx( 2nd [π] × alpha R ^ 2 ) STO alpha A ENTER ENTER

F3 2 "area is" , alpha A ENTER

To execute this program we must first, exit from programming mode by pressing **2nd** [QUIT]. To run the program, we enter **areacirc()** on the command line of the home screen and then press **ENTER**. Program **areacirc()** prompts the user with "radius". Upon entering 6, we obtain the display shown here.

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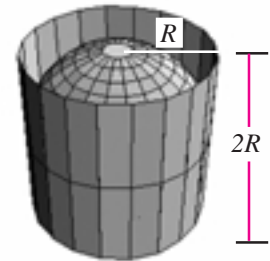


"THERE GOES ARCHIMEDES WITH HIS CONFOUNDED LEVER AGAIN"

Volume of the sphere is  $\frac{2}{3}$  volume of the cylinder.

$$V_{\text{sphere}} = \frac{4}{3}\pi R^3$$

$$V_{\text{cylinder}} = 2\pi R^3$$



#### Program on the Screen

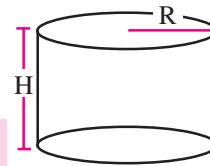
```
:areacirc()
:Prgm
:ClrIO
:Input "radius",r
:approx(π*r^2)→a
:Disp "area is",a
:EndPrgm
```

```
radius
6
area is
113.097
```

↑ The number of digits shown depends on the mode setting.

# WORKED EXAMPLES

The following example shows how we can embed one program in another.



## WORKED EXAMPLE 2

Use program **areacirc( )** (developed in example 1) to create a program, **volcylin( )**, to compute the volume of a right circular cylinder of height, H, and radius R.

Use **volcylin( )**, to compute the volumes of cylinders of these dimensions:

Height	3.6 cm	4.86 m	78.2 cm
Radius	1.4 cm	2.57 m	17.4 cm

### SOLUTION

Keying Sequence	Program on the Screen
<b>APPS</b> <b>7</b> <b>3</b> <b>▼</b> <b>▼</b> <b>volcylin</b> <b>ENTER</b> <b>ENTER</b> <b>▼</b> <b>▼</b>	<pre> :volcylin() :Prgm :ClrIO :Input "height",h :areacirc() :a*h→v :Disp "volume is",v :EndPrgm                     </pre>
<b>CATALOG</b> <b>ClrIO</b> <b>ENTER</b> <b>F3</b> <b>3</b> "height" <b>,</b> <b>alpha</b> <b>H</b> <b>ENTER</b>	
<b>areacirc( )</b> <b>ENTER</b> <b>alpha</b> <b>A</b> <b>×</b> <b>alpha</b> <b>H</b> <b>STO▶</b> <b>alpha</b> <b>V</b> <b>ENTER</b>	
<b>F3</b> <b>2</b> "volume is" <b>,</b> <b>alpha</b> <b>V</b> <b>ENTER</b>	

To execute this program, we must first exit from programming mode by pressing **2nd** [QUIT]. To run the program, we enter **volcylin( )** on the command line of the home screen and then press **ENTER**. Program **volcylin( )** prompts the user with "height" and then "radius". Upon entering 3.6 for the height and 1.4 for the radius, we obtain the display shown here.

```

radius
1.4
area is
6.15752
volume is
22.1671
    
```

**Note:** To apply **volcylin( )** to several sets of values, press **ESC** followed by **ENTER** after each run.

When creating a program, we can designate certain variables as *local* by pressing these keys **F4** **3** and listing the variables, separating them with commas. The values assigned to local variables are deleted after the program is run. See program **numfacto( )** p. 21.

When the remaining values of height and radius (given in the table above) are entered, the program **volcylin( )** displays respectively the volumes:  $V = 100.845$  and  $74,379.8$  (cubic units).

### DO WE REALLY NEED A PROGRAM TO CALCULATE THE VOLUME?

We could compute the volume for various values of the radius  $r$  and height  $h$  without using a program. We merely define the function  $vol(r,h) = \pi r^2 h$  as shown on the command line of this display.

```

Define vol(r,h)=π·r2·h
Done
define vol(r,h)=πr^2h
MAIN RAD AUTO SEQ
    
```

We can use **alpha** and the letter keys to enter the statement on the command line. To store this definition in memory, we press **ENTER**.

To calculate the volume of the cylinder with radius 1.4 and height 3.6, we enter the command shown in the display. Upon pressing **ENTER**, we obtain 22.1671 for the volume. One advantage to using a program is that we can define the variables  $r$ ,  $h$ , and  $v$  as local variables so that they are not stored in memory, leaving more variables free. Other advantages will emerge later.

```

Define vol(r,h)=π·r2·h
Done
Define vol(r,h)=π·r2·h
Done
Define vol(1.4,3.6) 22.1671
vol(1.4,3.6)
MAIN RAD AUTO SEQ 2/30
    
```

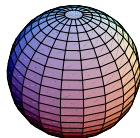
## EXERCISES & INVESTIGATIONS

1. What fraction of the volume of the tin shown in the diagram is occupied by the tennis balls? (Use the relationship discovered by Archimedes. (See page 14.)

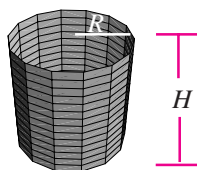


2. The volume of a sphere of radius  $R$  is given by

$$V_{\text{sphere}} = \frac{4}{3}\pi R^3.$$

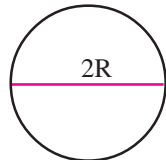


The volume of a cylinder of radius  $R$  and height  $H$ , is given by  $V_{\text{cylinder}} = \pi R^2 H$ .



Verify algebraically your answer to exercise 1.

3. The circumference  $C$  of a circle is given by  $C = 2\pi R$  where  $R$  denotes the radius. Write a program which prompts for the radius and then computes the circumference.

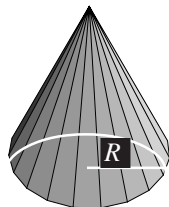


$$C = 2\pi R$$

4. The volume of a cone is one third the area of the base times the height.

a) Write a formula for the volume of a cone with circular base of radius,  $R$  and height,  $H$ .

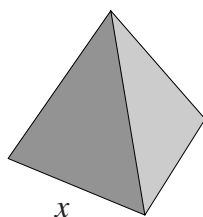
b) Write a program that prompts the user to enter the radius of the base of a cone and its height, and then computes and displays its volume.



5. The volume of a pyramid is one third the area of the base times the height.

a) Write a formula for the volume of a pyramid of height,  $h$ , with square base having side length  $x$ .

b) Write a program which prompts for the height and side length of the square base of a pyramid and then computes and displays its volume.



6. When it was completed around 2580 B.C., the Great Pyramid of Cheops (Khufu) was the tallest structure in the world. Its height was 147 m. Its base was a square with side length 230 m.

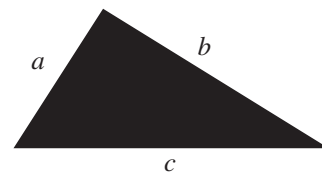
a) Use your program from exercise 5 to find the volume of the Pyramid of Cheops.

b) If all dimensions of the pyramid were increased by 10% what would be the percent increase in its volume?



7. Heron's formula for the area of a triangle with sides of length  $a$ ,  $b$ , and  $c$  is given by  $A = \sqrt{s(s-a)(s-b)(s-c)}$  where

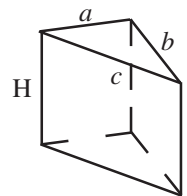
$$s = \frac{a+b+c}{2}.$$



a) Write a program which prompts for three sides of a triangle and displays its area,  $A$ .

b) Use your program to calculate the area of a triangle with sides 17 cm, 9 cm, and 21 cm.

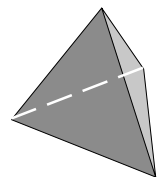
8. Write a program which prompts for the height of a triangular prism and the side lengths of its triangular base and then displays its volume. Note: The volume of any prism is the product of its height and the area of its base. (See exercise 7.)



9. Use the program you developed in exercise 8 to compute the volumes of the triangular prisms with these dimensions.

Height in cm	2.3	6.5	17
Sides of the base in cm	7.6, 5.1, 9	4.3, 3.7, 5	2.3, 7, 1.8

10. a) The volume of a pyramid is one-third the product of the area of its base and its height. Write a program which prompts for the height and the side lengths of the base of a triangular-based pyramid and yields its volume.



b) Use your program to find the volume of a pyramid of height 15 m with triangular base having sides of length 28 m, 35 m and 42 m.

## Solutions to Selected Exercises

### Exploration 3

1. If we assume that the balls are packed tightly in the tin so that sides of the tin is in contact with the balls, and that the balls are spherical and the tin cylindrical, then, as suggested by the Archimedes relationship, the tennis balls occupy two-thirds the volume inside the tin.

2. The combined volume of the three tennis balls would be:

$$3\left(\frac{4}{3}\right)\pi R^3 \text{ or } 4\pi R^3$$

The container which holds three balls of radius  $R$  has a base with area  $\pi R^2$  and height  $6R$ ; that is, a volume of  $6\pi R^3$ . Therefore the combined volume of the tennis balls is two-thirds the volume of the container.

3. The required program **circum( )** is shown below.

```
:circum()  
:Prgm  
:ClrIO  
:Local r,c  
:Input "radius",r  
:approx(2*pi*r)->c  
:Disp "circumference is",c  
:EndPrgm
```

4. a) The volume,  $V$  of a cone with circular base of radius,  $R$  and height,  $H$  is given by:

$$V = \frac{1}{3}\pi R^2 H$$

- b) The required program, **volcone( )**, is shown below.

```
:volcone()  
:Prgm  
:ClrIO  
:Local r,h,v  
:Input "radius",r  
:Input "height",h  
:approx(pi*r^2*h)/3->v  
:Disp "volume is",v  
:EndPrgm
```

5. a) The volume,  $V$ , of a pyramid with a square base of side length  $x$  units, and height  $h$ , is given by:

$$V = \frac{1}{3}x^2 h$$

- b) The required program, **volpyram( )**, is shown below.

```
:volpyram()  
:Prgm  
:ClrIO  
:Local x,h,v  
:Input "sidelength",x  
:Input "height",h  
:approx(x^2*h)/3->v  
:Disp "volume is",v  
:EndPrgm
```

Note that the quickest way to create **volpyram( )** is to recall program **volcone** from Exercise 4 and press F1 followed by 2 to save this program as **volpyram**. Open program **volpyram** and then modify it to match the program **volpyram( )** given above.