EXPLORATIONS ™

Objective

- Explain the probability of occurrence of compound events with and without replacement

Activity 4

To Replace or Not to Replace? That Is the Question

Introduction

Joshua keeps an opaque jar of candies on his desk. In the jar are four lemon sourballs and two cherry sourballs. Teasingly, he tells his sister Lisa that she can have a sourball as long as the one she takes is not his favorite. No matter which flavor she pulls out, he intends to say, "Oh, no! You can't have that one! It's my favorite flavor. Put that back and grab a different one!" He'll only allow her to have a sourball if the second is a different flavor.

Lisa sees right through her brother's ruse. Hoping to outwit her brother, she responds, "Well, I won't be able to remember what flavor I pulled the first time, so I'll hold on to the first sourball and if it is your favorite flavor, I'll pick a second one."

These two situations are very similar, yet there is an important distinction between them. In the first scenario, Lisa *puts back* the sourball she drew before drawing another. This represents a compound event *with replacement* so that the number of sourballs in the jar remains the same on each pick. In the second scenario, Lisa *keeps* the first sourball and then draws a second piece. This represents a compound event *without replacement* and the number of sourballs in the jar for the second draw is one fewer than the number of sourballs in the first draw.

Problem

Should Joshua allow Lisa to hold onto the first sourball while she picks a second one? In which situation, with replacement or without replacement, is Lisa more likely to get two sourballs of the same flavor? Would Lisa's chances of choosing the same flavor change if the jar had 40 lemon and 20 cherry sourballs?

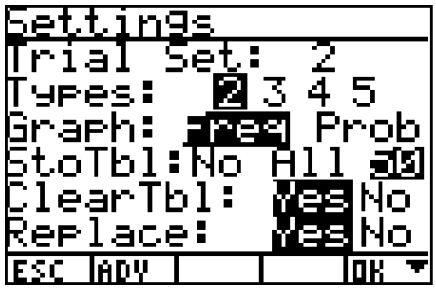
Exploration

Compound Event with Replacement

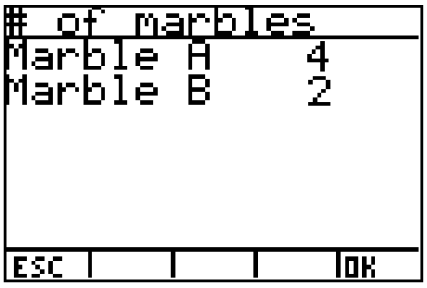
Respond to # 1 – 2 on the Student Worksheet.

1. Open the Probability Simulation application and select **Pick Marbles**.

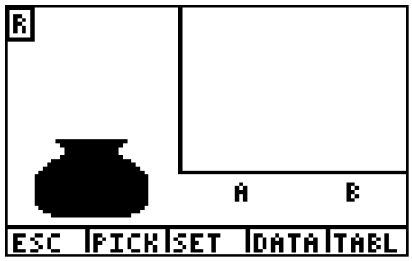
2. Select **SET** and then change the settings as shown.
Trial Set: 2 represents two draws. Types: 2 represents the two flavors of sourballs. Replace: Yes means that this simulation is conducted *with replacement* after each draw.



3. Select **ADV** and then set Marble A to **4** to represent the four lemon sourballs. Set Marble B to **2** to represent the two cherry sourballs.



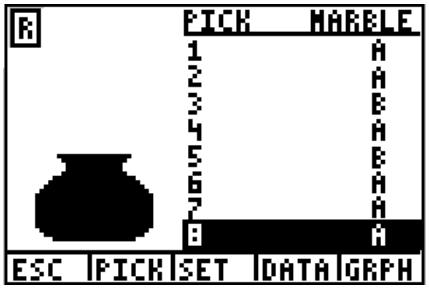
4. Select **OK** twice to return to the simulation screen.



5. Select **TABL** so that the data appears in table format.

6. Select **PICK.** This simulates choosing the sourball. You can select **PICK** again without clearing the table because the sourballs are being replaced each time. There are always four lemon and two cherry sourballs in the jar for every pick.

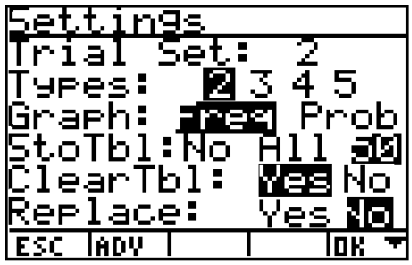
   In this exploration, a trial consists of two picks. The screen shows the results of four trials. Trials 1 and 4 have pick results that are the same.



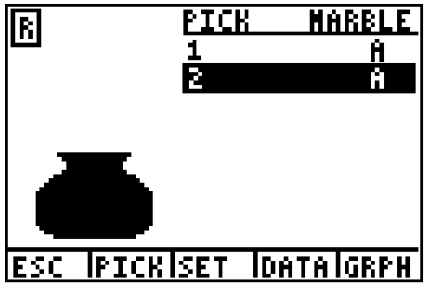
Respond to # 3 – 5 on the Student Worksheet.

Compound Event Without Replacement

1. In the second scenario, Lisa did not return the first sourball to the jar before picking the second one. To simulate this change, select **SET**, and then change Replace: to **No**.



2. Select **OK**, then select **YES** to return to the simulation screen.
3. Select **TABL** so that the data will appear in table format.
4. Before recording the data, you should look at how the change in the replacement setting affects the simulation. Select **PICK** a few times. You will run out of marbles after just three trials because all six of the marbles were picked!



5. To accurately carry out the simulation, you must clear the table after each trial. Select **SET** and change ClearTbl to **Yes**. Press ENTER, and then select **OK**.

   Note: *Before each trial (one pick of two marbles), make sure the table is empty. The table should never have more than two entries, as shown.*

Respond to # 6 – 11 on the Student Worksheet.

Extensions

You have been exploring the probabilities of situations with and without replacement by examining the results of the empirical data. Since the probabilities are very close, it may be hard to determine if the difference in the relative frequencies is due to chance variation or true differences without executing the simulation for thousands of trials. This can become very tedious. The extension question asks you to determine the theoretical probabilities of the various sourball scenarios.

Respond to Extension # 1 – 3 on the Student Worksheet.

Student Worksheet

Name _____

Date _____

To Replace or Not to Replace? That Is the Question

1. a. Without doing any calculations, what does your intuition suggest is the probability that Lisa will grab the same flavor on two picks when there are always four lemon and two cherry sourballs in the jar (that is, she replaces the first sourball drawn before choosing a second).

b. If, instead, there are 40 lemon and 20 cherry sourballs in the jar, how will the probability compare to your answer in #1a?

2. a. Without doing any calculations, what does your intuition suggest is the probability that Lisa will grab the same flavor on two picks when there are four lemon and two cherry sourballs in the jar and Lisa does NOT replace the first sourball before choosing the second?

b. If, instead, there are 40 lemon and 20 cherry sourballs in the jar, how will the probability compare to your answer in #2a?

3. Keep a tally for each pair of picks as to whether the sourballs are the same or different flavors. Record the results from 100 trials.

Sourball Table: 100 Trials with Replacement, 4 Lemon & 2 Cherry

	Same Flavors	Different Flavors
Tallies		
Total		
Relative Frequency		

4. Pool the class data for the sourball scenario with replacement. Record the results in the table below.

Total Trials	# of Trials with the Same Flavor	Relative Frequency of the Same Flavor

5. How does the relative frequency of the pooled class data compare with your intuition stated in #1a?

6. Keep a tally for each pair of picks as to whether the sourballs are the same or different flavors. Record the results from 100 trials

Sourball Table: 100 Trials with Replacement, 4 Lemon & 2 Cherry

	Same Flavors	Different Flavors
Tallies		
Total		
Relative Frequency		

7. Pool the class data for the sourball scenario with replacement. Record the results in the table below.

Total Trials	# of Trials with the Same Flavor	Relative Frequency of the Same Flavor

8. How does the relative frequency of the pooled class data compare with your intuition stated in #2a?

9. **a.** How does the relative frequency of drawing the same flavor change from the first to the second scenario?

b. In which situation is Lisa more likely to get two sourballs of the same flavor, with replacement or without replacement?

c. Should Joshua allow Lisa to hold onto the first sourball picked before choosing the second one?

10. The second question stated in the Problem section asks how the probabilities of each scenario would change if the sample size were increased to 40 lemon and 20 cherry sourballs - also a ratio of 2 to 1. Describe the design of a simulation using the Probability Simulation App to collect data that would help to answer this question. Make sure you include an appropriate number of trials.

11. For each scenario, how would the relative frequency of the outcomes change if the sample size were increased while still maintaining the same ratio of lemon to cherry sourballs.

Scenario 1 (with replacement):

Scenario 2 (without replacement):

Extensions

1. a. Determine the theoretical probabilities for the scenario of four lemon and two cherry sourballs with replacement. Show your work.

b. For 40 lemon and 20 cherry sourballs with replacement.

2. a. Determine the theoretical probabilities for the scenario of four lemon and two cherry sourballs without replacement. Show your work.

b. For 40 lemon and 20 cherry sourballs with replacement.

3. A jar is filled with 12 lemon sourballs and 9 cherry sourballs. Lisa wants a sourball. Joshua tells her that she can choose which challenge she wants to accept. Which one gives her the best probability of being able to keep a sourball? Explain.

Challenge 1: Reach into the jar and pull out a sourball, then put it back in. Reach in and pull out a second sourball. If the two picks are the same, she can keep the sourball.

Challenge 2: Reach into the jar and pull out a sourball, then put it back in. Reach in and pull out a second sourball. If the two picks are different she can keep the sourball.

Challenge 3: Reach into the jar and pull out two sourballs. If they are the same, she can keep one sourball and give the other to Joshua.

Challenge 4: Reach into the jar and pull out two sourballs. If they are different, she can keep one sourball and give the other to Joshua.

Teacher Notes

Activity 4

To Replace or Not to Replace? That is the Question

Objective

- Explain the probability of occurrence of compound events with and without replacement

Materials

- TI-84 Plus/TI-83 Plus

Teaching Time

- 90 minutes

Preparation

The students explore situations that involve compound events with and without replacement. The first simulation involves independent events since the item picked is returned to the jar. The second situation involves dependent events since the item picked is not returned.

To launch the activity, bring lemon and cherry sourballs to class and select students to act out the two scenarios described in the introduction. Next, take only a few minutes to have students respond to #1 and #2 on the Student Worksheet. Engage students in a class discussion to present and explain their intuitions. It is important that students hear a variety of rationales. Do not try to correct any misconceptions until after students have completed the simulations.

The theoretical probability for each scenario is nearly the same, so it is necessary to analyze data for a large number of trials. You should have at least 2000 trials in the pooled class data; otherwise, students may draw erroneous conclusions from their simulations.

After the simulation, the extension asks students to determine the theoretical probability. Depending on the background students have in probability, you may need to show students several ways to conceptualize the theoretical probability. One way is to list all possible combinations of selecting two sour balls, both with and without replacement. Summarizing these combinations in a probability tree diagram for compound events is helpful. A second way is to illustrate the problem with geometric area models.

Answers to the Student Worksheet

1. **a.** The intuitive probability a student predicts before running the simulation could easily be way off target. Estimates slightly greater than 0.5 are on target. Be alert, however, to false reasoning that suggests it will be exactly 0.5 since the sourballs will either match or not match. Also be alert to students who falsely reason it will be exactly 0.5 since the ratio of cherry to lemon is 0.5.

   b. The probability does not change as sourballs are added proportionately.

2. **a.** Focus the discussion on how their intuitive understandings about the subtle change of "no replacement" affects the resulting probabilities. Also, do not attempt to give correct answers at this point. If students think that the probability of comparing the two situations will differ, press them as to whether it will be greater or less and how much greater or less. Encourage them to give a rationale for their answers.

   b. As sourballs are added proportionately, the probability will increase and approach the probability with replacement.

3. Answers may vary widely.

4. Answers will vary, but will be the same within a class.

5. Answers will vary.

6. Answers will vary.

7. Answers will vary, but will be the same within a class.

8. Answers will vary.

9. **a.** Answers will vary depending on the variance in the experimental data. Relative frequencies for the first scenario should be slightly greater than 0.5, while in the second scenario, they should be slightly less than 0.5.

   b. Lisa is more likely to get two sourballs of the same flavor with replacement.

   c. No, assuming that Joshua does not want his sister to have one of his sourballs.

10. Answers will vary. The simulation would be very similar to that described in this activity. However, the number of marbles in the ADV setting of Pick Marbles would be changed to 40 and 20. The answer should indicate a design for a simulation with replacement, as well as one for without replacement. The design should suggest a minimum of 2,000 trials per simulation.

11. Answers will vary. Help students to realize that the theoretical probability remains the same in the first scenario. This can be explained by the fact that the chance of getting a lemon sourball on any one pick is always the same: 2 out of 3, or 0.667.

For scenario two, the theoretical probability increases as the sample size increases. The increasing pattern may not be apparent if not enough trials are simulated. Help students to understand that as the sample size increases, the ratio of choosing a lemon sourball on the second pick increasingly approaches that of a pick with replacement.

Answers to Extensions

1. **a.** The theoretical probability of selecting two sourballs that are the same flavor is the sum of the probability of selecting two lemon sourballs and the probability of selecting two cherry sourballs. The probability of selecting two lemon sourballs (with replacement) is $\frac{2}{3} \times \frac{2}{3}$ or $\frac{4}{9}$. The probability of selecting two cherry sourballs (with replacement) is $\frac{1}{3} \times \frac{1}{3}$ or $\frac{1}{9}$. The sum of the probabilities is 5/9 or approximately 0.556.

b. $\left(\frac{40}{60} \times \frac{40}{60}\right) + \left(\frac{20}{60} \times \frac{20}{60}\right) = \frac{5}{9}$ or approximately 0.556.

2. **a.** The theoretical probability of selecting two sourballs that are the same flavor is the sum of the probability of selecting two lemon sourballs and the probability of selecting two cherry sourballs. The probability of selecting two lemon sourballs (without replacement) is $\left(\frac{4}{6} \times \frac{3}{5}\right)$ or $\frac{6}{15}$. The probability of selecting two cherry sourballs (without replacement) is $\left(\frac{2}{6} \times \frac{1}{5}\right)$ or $\frac{1}{15}$. The sum of the probabilities is $\frac{7}{15}$ or approximately 0.467.

b. $\left(\frac{40}{60} \times \frac{39}{59}\right) + \left(\frac{20}{60} \times \frac{19}{59}\right) = \frac{97}{177}$ or approximately 0.548.

3. Challenge 4 gives the best chance. The probabilities for each challenge are (1) 0.510, (2) 0.490, (3) 0.486, and (4) 0.514.