Name: _____ Group Members: _____

Exploration 3-6a: Derivative of the Sine of a Function

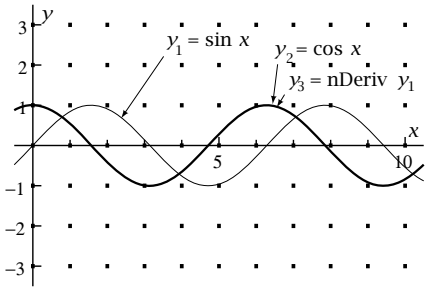
Date: _____

Objective: Find the derivative of the sine function if the argument is a *function* of x.

The figure shows the graphs of

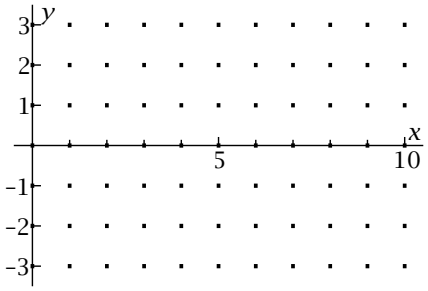
$y_1 = \sin x$
$y_2 = \cos x$
y_3 = numerical derivative of y_1 (thick style)

As you can see, there is strong graphical evidence that the derivative of $\sin x$ is $\cos x$. Assuming that this is true, you are to investigate derivatives of **composite functions** of the form $y = \sin$ (function of x).



1. Duplicate this figure on your grapher. Use the window shown. Turn on the grid to help you see the critical points. Does the derivative of $\sin x$ really seem to be $\cos x$? _____

2. Let $g(x) = \sin 3x$. Change y_1 to $\sin 3x$. Deactivate y_2 and y_3. Plot the graph of y_1 and sketch the result here.



3. Without actually plotting the graphs, make a conjecture about what function $g'(x)$ equals.

   Conjecture: $g'(x) =$ _____

4. Plot the three graphs

   $y_1 = \sin 3x$
   y_2 = your conjecture in Problem 3
   y_3 = numerical derivative of y_1

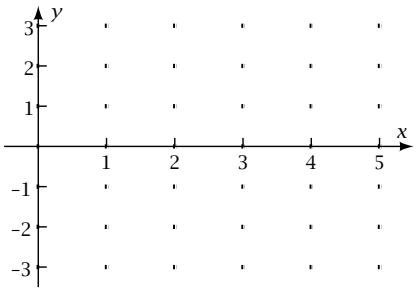
   Was your conjecture correct? If so, indicate this fact. If not, change your conjecture and replot the graphs until your conjecture matches the numerical derivative graph. Write the final answer here.

   $g(x) = \sin 3x$

   Actual $g'(x) =$ _____

   Was your original $g'(x)$ conjecture correct? _____

5. Enter $y_1 = h(x) = \sin x^2$ on your grapher. Deactivate y_2 and y_3. Change the window as shown here. Sketch the resulting graph.



6. Make a conjecture about what an equation for $h'(x)$ might be. Then verify (or refute!) your conjecture by appropriate graphing on your grapher. If your conjecture was wrong, figure out a correct equation for $h'(x)$.

   $h(x) = \sin x^2$

   Original conjecture: $h'(x) =$ _____

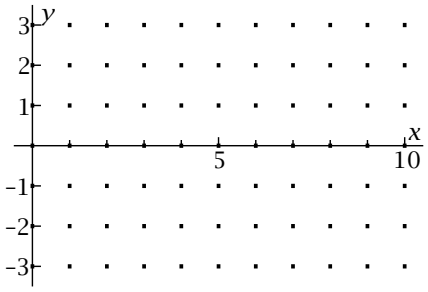
   Final answer: $h'(x) =$ _____

7. What operation could you perform on the original equation, $h(x) = \sin x^2$, to get the actual derivative you found in Problem 6?

8. Reset the window as in Problem 1. Plot the graph of $y_1 = t(x) = \sin x^{0.7}$. Sketch the result below. Make conjectures about the derivative function until you have found an equation for y_2 that matches the derivative graph in y_3. Write the result here.

   $t(x) = \sin x^{0.7}$

   $t'(x) =$ _____



9. Look up the **chain rule** in your text. On the back of this sheet, tell what you learned about how to find the derivative of a composite function using the chain rule.

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