# Nonlinear Systems of Equations 

## Activity Overview

This activity is designed to be used an as introduction to nonlinear systems of equations. It begins by allowing students to move figures around the screen to see ways certain types of graphs (linear/conic and conic/conic) can intersect each other and how many possible intersection points are possible. The activity concludes by having students look at the equations in a nonlinear system, stating how many solutions are possible, and then solving the system by graphing.

Topic: Conics

- Verify the solution to a linear-quadratic system of equations by graphing and using Intersection Point(s).


## Teacher Preparation and Notes

- This activity is designed to be used in a Precalculus classroom. It can also be used in an Algebra 2 classroom. This activity is intended to be mainly teacher-led, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.
- Students should be familiar with the graphs and equations of the four conic sections: parabolas, circles, ellipses, and hyperbolas.
- Students should also understand transformations of parabolas, specifically how the sign of the squared term indicates whether the parabola opens up or down, as well as how to translate a parabola vertically.
- Notes for using the TI-Nspire ${ }^{\text {TM }}$ Navigator ${ }^{\text {TM }}$ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9982" in the keyword search box.


## Associated Materials

- NonlinearSysEqns_Student.doc
- NonlinearSysEqns.tns
- NonlinearSysEqns_Soln.tns


## Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- How Many Solutions (TI-Nspire technology) - 9284


## Problem 1 - Numbers of possible intersection points

If needed, define nonlinear system of equations. The system is one in which at least one equation is nonlinear. As with a linear system, the solutions of the system correspond to the intersection points of the graphs of the equations in the system.

On page 1.2, students are directed to move the circle around by its center to determine how many intersection points could exist between a line and a circle. They should do the same on page 1.3 for a line and a parabola.
Students should conjecture that for the graphs of a linear function and conic section, there are either 0 , 1 , or 2 points of intersection.

Allow students time to test their conjecture with the graphs on page 1.5.
To move between screens, press tatrl tab.


In how many ways can a line and circle intersect? Move point $A$ to find out.


Now students will look at the intersections of pairs of conic sections. On page 1.7, students will see a circle and hyperbola with no intersection points.
Students should animate point $A$ by choosing MENU > Actions > Attributes, selecting point $A$, moving down to the second attribute (which shows a speed of 0 for the animation), and typing a nonzero speed, such as 1 or 2 (the greater the number, the faster the animation).

Students will watch as the circle moves across the screen. They can stop the motion at any time by pressing 0 . They will see the circle intersect the hyperbola at 2,3 , and 4 places.

Ask students if they think it is possible for a circle to intersect a hyperbola at just one point.


On page 1.9, students are instructed to create a circle (MENU > Shapes > Circle) anywhere on the page. They should move it around the screen and find the different situations in which it intersects the hyperbola exactly one time.
Note: When students create the circle, make sure that they do not place the center on one of the axes.


## TI-Nspire Navigator Opportunity: Screen Capture

## See Note 1 at the end of this lesson.

Page 1.11 shows a parabola and hyperbola with no intersection points. Students should move the parabola around to see how many points of intersection with the hyperbola are possible.

To move the entire parabola without changing its width, student should move the cursor near its vertex until the cursor looks like this: $\ddagger$.

Students should make a conjecture about the number of intersections between the graphs of two
 conic sections. (There can be $0,1,2,3$, or 4 intersection points.)

## TI-Nspire Navigator Opportunity: Screen Capture

See Note 2 at the end of this lesson.

## Problem 2 - Two parabolas

Problem 2 challenges students to make graphs to show the ways in which two parabolas can intersect. On page 2.2, students are to construct graphs to show two parabolas not intersecting.

Students can use the function Entry Line at the bottom of the screen, or select MENU > Actions > Text, display the function rule, and drag it to the $x$-axis.


Allow students time to work independently on pages 2.3 and 2.4 , making parabolas that intersect in exactly $1,2,3$, and 4 places.
If students have difficulty with page 2.4 , remind them that parabolas could open to the left or to the right. Remind them that such a parabola is not a function, and that they must define two functions to graph this type of parabola.

This part of the activity is spatial, with the focus on how the parabolas can intersect. It is not a place for students to get bogged down in calculations. Therefore, when students are constructing parabolas with three intersection points, it is sufficient for students to first make the parabola opening left or right, and then drag a parabola opening up or down until it appears that they intersect at exactly three points.



Problem 3 - Solving nonlinear systems by graphing
This problem consists of four systems of equations to solve. For each, students should first decide how many solutions are possible. Then, they find the approximate solutions by graphing the equations and finding the intersection points (MENU > Points \& Lines > Intersection Point(s)).
Students may need to change the Window
Settings to view all the intersection points. Walk around the room and assist students as needed.

4. $x^{2}+y^{2}=15$ and $y+6=x^{2}$


Students who finish early can solve the systems algebraically by using elimination or substitution.

## TI-Nspire Navigator Opportunity: Screen Capture and Quick Poll

See Note 3 at the end of this lesson.

TI-Nspire Navigator Opportunities

## Note 1

## Problem 1, Screen Capture

Use Screen capture to display student screens showing the systems intersecting in exactly one location. Discuss how there are many solutions to this problem as displayed by the many different screens in the Screen Capture.

Note 2
Problem 1, Screen Capture
Use Screen Capture to collect student screens showing o, 1, 2, 3 and 4 intersections.

## Note 3

Problem 3, Screen Capture, Quick Poll
Use Screen Capture to monitor student progress as they work through the four problems on pages 3.1-3.5. Send Quick Polls to gather student responses for the intersections points for each question.

