NUMB3RS Activity: Growing Geometrically
Episode: "Identity Crisis"

Topic: Geometric sequences **Grade Level:** 9 - 12
Objective: To learn explicit formula for *n*th term of a geometric sequence.
Materials: TI-83 Plus/TI-84 Plus graphing calculator
Time: 15 - 20 minutes

Introduction

In "Identity Crisis," Charlie learns about a scheme to defraud people by removing money from their bank accounts. The perpetrator removes the money and later replaces it with money taken from twice as many accounts, keeping the extra money. This technique eventually catches up with the perpetrator. This is an example of a geometric sequence with a ratio of 2. In this activity, students will use that same ratio to do an allowance problem and then use some other ratios to grasp the concept of geometric progressions and series.

Discuss with Students

Point out to the students that the terms "geometric sequence" and "geometric progression" are used interchangeably. If this is not something students have seen before, explain how the "first term" might not always be obvious. In question 6, the $100 is deposited at the beginning (hence year zero). In question 7, the question is "after each bounce," so that the "first" term is after the first bounce. These examples are to encourage class discussion. The teacher should moderate the discussion to ensure that students will not get confused when they meet similar problems on standardized tests.

A geometric sequence is in the form $a_n = a_1 r^{n-1}$. The exponent ($n - 1$) will equal zero for $n = 1$, so remind students that $r^0 = 1$ for $r \neq 0$.

Student Page Answers:
1. ar^{n-1} **2.** *512; 52,4288* **3.** *$5,368,709.12* **4a.** *$100,000 (which comes from a = 10, r = 10 and n = 5 so* $10(10^{5-1}) = 10(10^4) = 10^5$*)* **4b.** *Answers will vary, but in just 9 iterations, it is already at 1,000,000,000 people, and in the 10th iteration, it is more than the total number of people on the planet. This is the basic problem that Charlie points out on the show.* **5.** *$814.97, as long as all 43 years have passed.* **6.** *5.9049 m*

Bro. Patrick Carney, DePaul Catholic High School, Wayne, NJ

Name: ________________________________ Date: ________________

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A geometric sequence is one in which each term of the sequence is obtained by multiplying the previous term by a certain fixed number (frequently called the "ratio," because it is the ratio of any two consecutive terms).

For example, in a geometric sequence where the first term is a, the second is ar, the third term is ar^2, the fourth term is ar^3, etc.

This sequence can be written as a, ar, ar^2, ar^3, …

1. What is the nth term of the sequence in the example above?

2. In the case Charlie deals with, the first term is 1 and the ratio is 2. This sequence is 1, 2, 4, 8, 16, … How many bank accounts would be needed for the 10th term? How many for the 20th term?

3. A similar problem is informally called the allowance problem. Supposedly, parents are negotiating an allowance with their child, who suggests that the parents pay 1 cent on the first day of the month, 2 cents on the second day, 4 cents on the third day, etc. The allowances paid on each day of the first week are shown below.

Day	Amount Paid on That Day
1	$0.01
2	$0.02
3	$0.04
4	$0.08
5	$0.16
6	$0.32
7	$0.64

Use your calculator and the answer to Question 1 to find the amount that would be paid on the 30th day.

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4. Another form of a geometric sequence is a chain letter scheme. These were popular via the postal system in previous years and have made a comeback on the Internet. A typical one might be that a person receives a letter with a list of 5 names and is told to send $1.00 to the name at the top of the list, make 10 copies of the letter with his or her name at the bottom, and remove the name of the person to whom $1.00 was sent. These 10 copies are sent to 10 different people, and the promise is that when one's name reaches the top of the list, a certain amount of money will be received.

a. Using what you learned about geometric sequences, determine how much money is promised.

b. Explain the flaw in the reasoning that all participants would get rich by this scheme.

5. Suppose that, after finishing college, a graduate who just turned 22 deposits $100 in the bank and leaves it untouched until retirement at age 65. Assume the bank pays 5% interest annually. If no other withdrawals or deposits are made, how much money would be in that bank account at that time? (**Hint:** Use $r = 1.05$ because the account keeps the initial deposit (multiplies by 1) and then adds 5% of that money (0.05 times the amount on deposit). Thus the rate is the sum of these, or 1.05.)

6. Geometric sequences do not always get larger. If the absolute value of the ratio is less than 1, the absolute value of each term gets smaller. For example, suppose a ball is dropped and is at a height of 10 m after the first bounce. If each bounce is 90% of the height of the previous high position, what is the height of the ball after the fifth bounce?

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Geometric sequences appear in many of the other courses studied in school. Find some applications in other courses you study. For example, in biology, cells splitting results in a geometric sequence; in social studies, population growth can be a complicated application of the principle; and in economics, an expansion rate might be a geometric sequence.

Additional Resources

For more information on Geometric Sequences, visit the following Web sites:

- **http://mathworld.wolfram.com/GeometricSequence.html**
- **http://myweb.cwpost.liu.edu/arockett/Mth1-Geo.PDF**
- **http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut54d_geom.htm**

For the Student

A classic problem involves the legend of the person who invented chess. The king was so impressed, he offered to give the inventor whatever he wanted. The inventor asked for a grain of wheat on the first square of the chess board, 2 on the second, 4 on the third, etc. The king supposedly said "Thou fool, I would have given you half my kingdom and you only asked for this!" In fact, figure out what he actually requested. For example, calculate the total number of grains when the 64 squares are filled, multiply by the weight of one grain and compare that to the annual grain output of the world today. For an alternative exercise, find the area of land for the entire Earth and calculate how many grains would be in each square foot if the grain was evenly distributed over each square inch of dry land.

Related Topic

Another kind of sequence is the arithmetic sequence (progression), in which each term is formed by adding a constant to the previous term. A finite arithmetic series is the sum of a finite number of terms of the sequence. See **http://www.ltcconline.net/greenl/courses/154/seqser/aritmet.htm** for some examples.

Bro. Patrick Carney, DePaul Catholic High School, Wayne, NJ