Math Objectives

- Students will be able to prove that when two pairs of corresponding angles of two triangles are congruent, the third pair of angles will be congruent.
- Students will recognize and apply the Angle-Angle Similarity Theorem.
- Students will attend to precision (CCSS Mathematical Practice).

Vocabulary

- proportional
- similarity
- congruence
- corresponding parts

About the Lesson

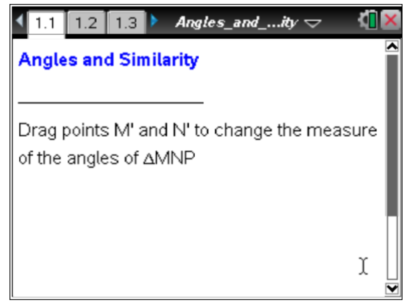
- This lesson involves investigating the relationship among angles in similar triangles.
- As a result, students will:
- Change the angle measures of one triangle to match the angle measures in another triangle.
- Prove that when two pairs of corresponding angles of two triangles are congruent, the third pair of angles will be congruent.
- See the equal ratios of the sides when two angle measures match showing that the congruence of two pairs of corresponding angles is sufficient to make two similar triangles.
- Apply the ratio of the sides of similar triangles to find the measure of one side when they know the measure of a given corresponding side.

TI-Nspire™ Navigator™

- Use Class Capture to formally assess students' understanding.
- Use Quick Poll to assess students' understanding.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity

- Angles_and_Similarity_Student.pdf
- Angles_and_Similarity_Student.doc

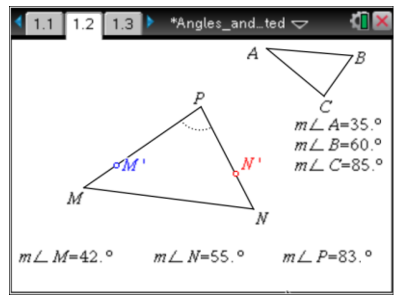
TI-Nspire document

- Angles_and_Similarity.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor (arrow) until it becomes a hand () getting ready to grab the point. Press ctrl to grab the point and close the hand (). When finished moving the point, press esc to release the point. Note: When changing the measure of angles *M* and *N*, it may be necessary to move points *M*' or *N*' in multiple directions to get the desired angle measure.

Move to page 1.2.



1. Drag points *M′* and *N′* until $m\measuredangle M = m\measuredangle A$ and $m\measuredangle N = m\measuredangle B$. What changes when you move these points?

   <u>Answer:</u> *M′* changes $\measuredangle M$ and $\measuredangle P$, and point *P* moves changing the lengths of $\overline{NP}$ and $\overline{MP}$.
   Moving *N′* changes $\measuredangle N$ and $\measuredangle P$, and point *P* moves changing the lengths of $\overline{NP}$ and $\overline{MP}$.

Teacher Tip: The emphasis in this activity will focus on moving *M*' and *N*' to change the angle measures of $\triangle MNP$. Students will not be measuring segments.

2. Drag points *M′* and *N′* until $m\measuredangle M = m\measuredangle A$ and $m\measuredangle N = m\measuredangle B$.
   a. Record the angle measures for each of the angles below:
   $m\measuredangle M$ = ___ $m\measuredangle N$ = ____ $m\measuredangle P$ =____

   <u>Answer:</u> $m\measuredangle M$ is 35°, $m\measuredangle N$ is 60°, and $m\measuredangle P$ is 85°

   b. What do you notice about the relationship of $m\measuredangle P$ and $m\measuredangle C$?

   <u>Answer:</u> They both measure 85°.

Teacher Tip: Students may note that the three angles of a triangle must add up to 180°. If not noted here, it will be addressed in question 4.

3. a. Drag points M' and N' until $\measuredangle M$ and $\measuredangle N$ are congruent to two corresponding angles in ΔABC that are different than the angles identified in question 2a. Record the angle congruence for each of the angles below:

$m\measuredangle M$ = _____ $m\measuredangle N$ = _____ $m\measuredangle P$ = _____

Answer: For example, if $m\measuredangle M = 85°$, $m\measuredangle \text{N} = 60°$, and $m\measuredangle P = 35°$, then $\measuredangle M \cong \measuredangle C$, $\measuredangle N \cong \measuredangle B$, and $\measuredangle P \cong \measuredangle A$.

b. How many different ways can this be done? List any other possibilities and justify your answer.

Answer: There are six cases. There are two cases where $\measuredangle N \cong \measuredangle B$. Either $\measuredangle M \cong \measuredangle C$ and $\measuredangle P \cong \measuredangle A$ or vice versa. Similarly, there are two cases where $\measuredangle N \cong \measuredangle A$ and two cases where $\measuredangle N \cong \measuredangle C$.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

4. Keep in mind that two angles are **congruent** if and only if they have the same measure.

If two triangles have two pairs of angles that are congruent, what can you conclude about the third pair of angles? Give a mathematical argument for your conclusion.

Answer: The third pair of angles must be congruent. The sum of the measures of the interior angles of a triangle is 180°. This means that $180° = m\measuredangle M + m\measuredangle N + m\measuredangle P = m\measuredangle C + m\measuredangle B + m\measuredangle A$. If $\measuredangle M \cong \measuredangle C$ and $\measuredangle N \cong \measuredangle B$, then $m\measuredangle M + m\measuredangle N + m\measuredangle P = m\measuredangle M + m\measuredangle N + m\measuredangle A$. Then $m\measuredangle P = m\measuredangle A$ and $\measuredangle P \cong \measuredangle A$.

Teacher Tip: Students may use specific angle measures to make this argument. Guide students to a more general argument.

5. Why would your triangles in question 3 be described as similar, but not congruent?

Answer: The two triangles are similar because pairs of corresponding angles are congruent. The triangles are not congruent because their corresponding sides are not congruent.

Teacher Tip: This question asks students to remember the definition of similar figures. (Corresponding angles are congruent and corresponding sides are proportional.) If students say the triangles have the same shape, emphasize that similarity is determined by congruent angles, not how a shape looks.

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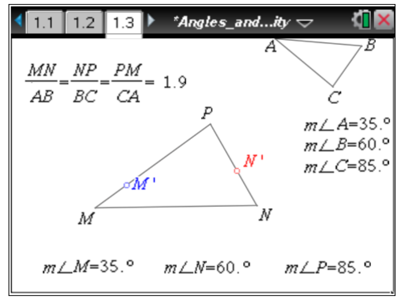


6. Drag points *M′* and *N′* until $\angle M \cong \angle A$ and $\angle N \cong \angle B$.

   a. What additional information is given when the corresponding angle measures are equal to each other?

   Answer: A proportion appears, and it is approximately 1.9.

> **Teacher Tip:** The answers here are all approximations. The angle measurements are rounded on the TI-Nspire. **Answers may vary slightly based on rounding error.**

   b. What is significant about this information?

   Answer: If the triangles are similar, with $\overline{MN}$ corresponding to $\overline{AB}$, then $\overline{MN}$ is 1.9 times $\overline{AB}$. $\overline{NP}$ to $\overline{BC}$ and $\overline{PM}$ to $\overline{CA}$ have the same ratios.

   c. After completing questions 6a and 6b, Frank concludes that $\Delta ABC \sim \Delta NPM$. Is his reasoning correct? Explain why or why not.

   Answer: He is wrong. The two triangles are similar, but the similarity statement should read $\Delta ABC \sim \Delta MNP$ to reflect corresponding congruent angles.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

7. Knowing two pairs of corresponding angles are congruent leads to proportional sides. When you know the ratio of similarity and a side in one triangle, you can use it to find the measure of the corresponding side in the other triangle.

   a. Drag *M′* and *N′* until $m\angle M = 35°$ and $m\angle N = 60°$.

   What is the ratio of similarity shown on the sketch?

   Answer: 1.9

   Which side of ΔABC corresponds to $\overline{MN}$?

   Answer: $\overline{AB}$

If $\overline{MN} = 8$, find the length of the corresponding side. Show your work.

Answer: $\frac{8}{AB} = 1.9$; $AB \approx 4.2$

b. Drag M' and N' until $m\angle M = 35°$ and $m\angle N = 85°$.

What is the ratio of similarity shown on the sketch?

Answer: 2.5

Which side of ΔMNP corresponds to $\overline{BC}$?

Answer: $\overline{PN}$

If $BC = 2$, find the length of the corresponding side. Show your work.

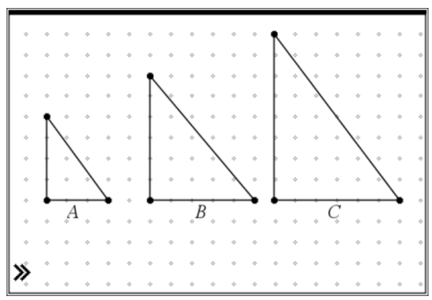
Answer: $\frac{PN}{2} = 2.5$ **;** $PN = 5.0$

Teacher Tip: The answers here are all approximations. The angle measurements are rounded on the TI-Nspire. **Answers may vary slightly based on rounding error.**

TI-Nspire Navigator Opportunity: Quick Poll
See Note 3 at the end of this lesson.

8. Terry and Jessica created three right triangles shown on the screen on the right. Terry claims that all the right triangles are similar because they all have a right angle.

Jessica says that you also need to know that the measures of a pair of corresponding acute angles are congruent before you can conclude that any two of the right triangles are similar.



Teacher Tip: Triangle *A* is similar to Triangle *C*. You can establish this with the right angle and the side measures from grid: $\frac{4}{8} = \frac{3}{6}$ (Note that this uses the SAS theorem of similar triangles—a theorem not covered in this activity). The sides of Triangle *A* have been doubled to get Triangle *C*. To create Triangle *B*, 2 units were added to each leg of Triangle *A*—some students mistakenly think you can add the same value to the sides of triangle to create a similar triangle. You may choose to have this discussion if your students have the prerequisite knowledge of all the similarity theorems.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent and the triangles are similar.
- If you know that two triangles have two pairs of congruent corresponding angles and you are given the ratio of two corresponding sides, then you can find the measure of a side in one triangle given the measure of its corresponding side in the other triangle.

Assessment

Right triangle *ABC* has one acute angle that is 55°. Right triangle *DEF* has one acute angle that is 35°. Are the two triangles similar?

Answer: The triangles are similar since the angles in triangle *ABC* are congruent to the angles in triangle *DEF*. However, you cannot write a similarity statement without knowing specifically which angles are right and which measure 55° and 35°.

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Note 1

Question 3, *Class Capture*: Since there are six different ways that students can move *M*' and *N*' in order for the angles in the two triangles to be congruent, use Class Capture to view how students have moved the points. You might sort the screens by case.

Note 2

Question 6c, *Quick Poll*: Send students an Agree/Disagree Quick Poll to determine whether they agree or disagree with Frank. Have students share their reasoning with the class.

Note 3

Question 7, *Quick Poll*: Send students an Open Response Quick Poll to collect their responses to the calculated side length.