TI-*nspire*™

Complex Numbers: Plotting and Polar Form

by – Sean Turkington

Activity overview

This activity is designed for students who have had prior experience with complex numbers. They first refresh their memories of basic operations with complex numbers. Students then learn to plot complex numbers. Students learn the basics of writing complex numbers in their polar forms and comparing them to their rectangular form. Finally, students plot complex numbers in polar form by entering their modulus and arguments into a spreadsheet linked to a Graphs & Geometry page.

Concepts

Complex numbers, polar form of complex numbers, modulus and argument of complex numbers, plotting complex numbers

Teacher preparation

- *This is designed to be a self-guided walk-through of plotting complex numbers, finding modulus and arguments of complex numbers, and converting complex numbers to their polar forms.*
- *Teacher must transfer to student handhelds the .tns file before beginning the activity.*
- *Teacher should make sure the students are familiar with the location of the **i** key.*

Classroom management tips

*During the activity the teacher should circulate and answer questions as necessary. Most of the activity is pre-set but students will need to locate the **i** key.*
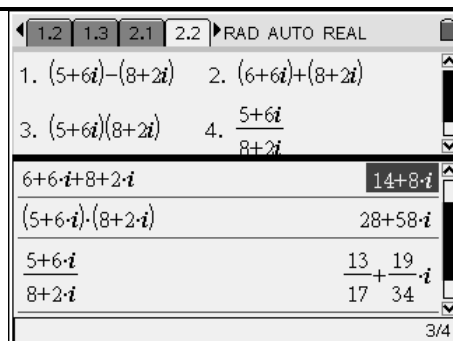
TI-Nspire Applications

Notes, Calculator, Graphs & Geometry

Step-by-step directions

Problem 1 is a review of the basic concept of a complex number. Page 1.2 defines complex numbers. On page 1.3 students are asked to find the values of *a* and *b* for several different complex numbers in a spreadsheet. Correct student responses are shown in the screen shot to the right.

Problem 2 asks students to review the operations of adding, subtracting, multiplying, and dividing complex numbers. Students might need a refresher on multiplication and division. It is assumed they have previous experience with these operations. On page 2.2 students use the Calculator application to confirm answers they find on paper.
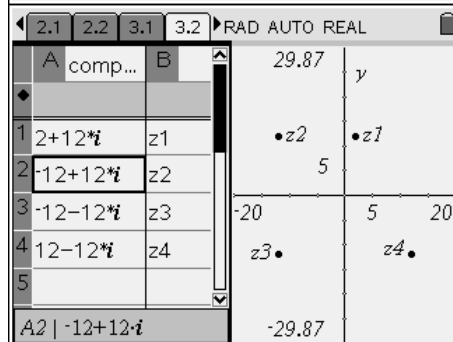
Problem 3 describes what it means to plot a complex number. Students almost universally accept this idea. Page 3.1 describes the difference between the real coordinate system and the complex coordinate system. Page 3.2 is a split screen of a Spreadsheet and Graphs & Geometry. In the spreadsheet are four complex numbers, $z1$, $z2$, $z3$, $z4$, one each in the four quadrants. Students are asked to change the complex numbers and watch as their plotted values move around on the Graphs & Geometry page. To change the complex number you must be highlighting the cell and then press the enter key. Students should spend as much time on this page as they need until they understand how the complex numbers are being plotted.
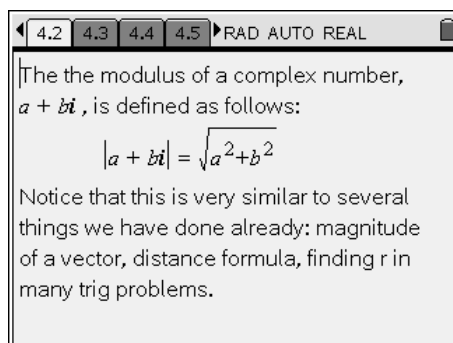
Problem 4 begins the process of converting complex numbers from rectangular to polar form. Page 4.2 explains what the modulus of a complex number is, shows the proper notation for it, and how to calculate it. This page also references several similar calculations students have done in the past.
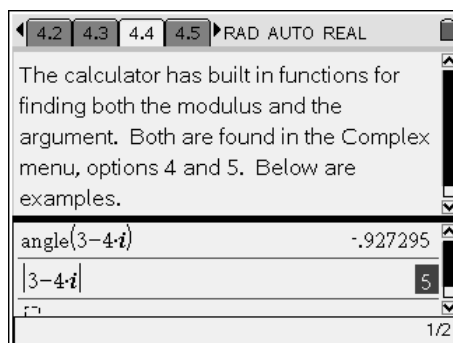
Page 4.3 explains what the argument of a complex number is and how to calculate it by using arctangent and a cases approach.

Page 4.4 shows how to use the built-in calculator functions to find the argument and modulus. Polar angle and polar radius are both found under the complex menu of the Calculator application and in the Complex menu of the Catalogue. Students should be very aware at this point of what mode (degree or radian) their handheld is in.


Page 4.5 formally introduces the polar form of a complex number. It also introduces the common short-hand expression rcis(t) notation. On page 4.6 students practice converting a complex number to polar form using the handheld. Sample student work is shown at right.

Problem 5 is a hint of what is to come with polar form of complex numbers. Page 5.2 is set up as a Spreadsheet/Graphs & Geometry split. Students are asked to enter r and theta and the point is plotted. Students typically enjoy this and enjoy trying to arrange the points as the vertices of regular polygons.
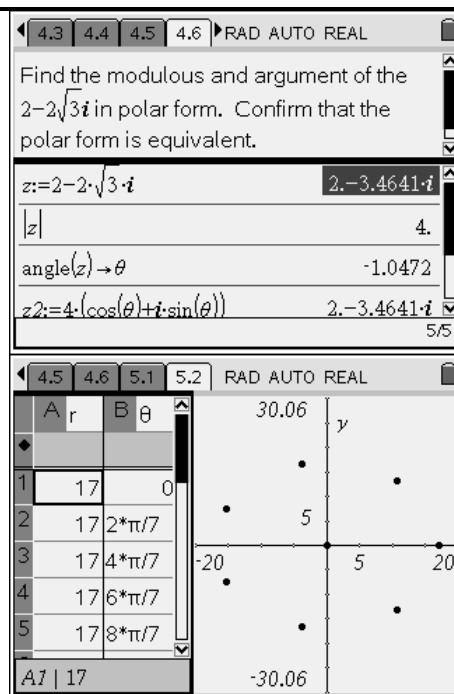
Assessment and evaluation

- *At the completion of this activity students will be able to describe the basics of converting a complex number to polar form. They should be able to compare it to other, very similar, procedures they have encountered before. This makes an excellent journal entry.*

Activity extensions

- *Ask students to arrange the complex numbers so that they are the vertices of a particular n-gon.*

  Students should be able to generalize the process of determine necessary values of theta for any n-gon.

Student TI-Nspire Document

Filename goes here.

Complex Numbers: Plotting and Polar Form
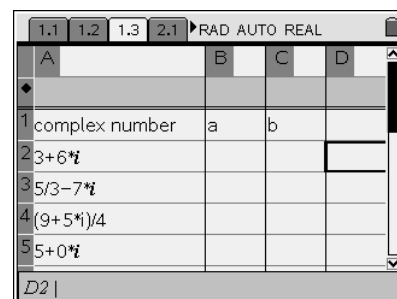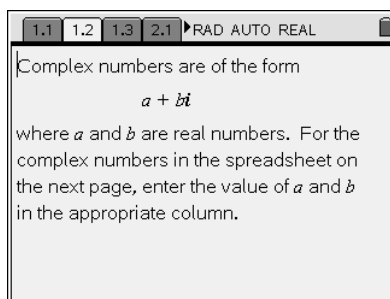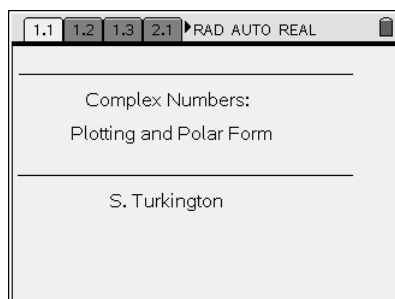
| 1.1 | 1.2 | 1.3 | 2.1 | ▶RAD AUTO REAL |

In previous courses you have studied imaginary numbers and some operations on complex numbers. It is expected that you know how to add, subtract, multiply, and divide complex numbers. On the next page use the calculator to complete the indicated operations. Verify that you can do these operations by hand.

| 1.2 | 1.3 | 2.1 | 2.2 | ▶RAD AUTO REAL |

1. $(5+6i)-(8+2i)$ 2. $(6+6i)+(8+2i)$

3. $(5+6i)(8+2i)$ 4. $\dfrac{5+6i}{8+2i}$

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| 1.3 | 2.1 | 2.2 | 3.1 | ▶RAD AUTO REAL |

Plotting complex numbers uses a method similar to plotting points in Cartesian coordiantes. The horizontal axis is the real number line, the vertical axis is the complex number line, and the origin is the intersection of the two at $0 + 0i$. Change the complex numbers in Column A to better understand how plotting complex numbers works.

| 2.1 | 2.2 | 3.1 | 3.2 | ▶RAD AUTO REAL |

	A comp...	B
◆		
1	12+12*i	z1
2	-12+12*i	z2
3	-12-12*i	z3
4	12-12*i	z4
5		

A | complex

29.87
y

•z2   •z1
      5
-20   5   20

z3•   z4•
-29.87

| 2.2 | 3.1 | 3.2 | 4.1 | ▶RAD AUTO REAL |

A common way of expressing complex numbers is in polar form. To do this we need to find two attributes of the number: its modulus (or absolute value) and its argument (or direction angle).

| 3.1 | 3.2 | 4.1 | 4.2 | ▶RAD AUTO REAL |

The the modulus of a complex number, $a + bi$, is defined as follows:

$$|a + bi| = \sqrt{a^2+b^2}$$

Notice that this is very similar to several things we have done already: magnitude of a vector, distance formula, finding r in many trig problems.

| 3.2 | 4.1 | 4.2 | 4.3 | ▶RAD AUTO REAL |

The argument of a complex number, $a +bi$, can be found using arctangent and a number of cases.

QI: $\theta = \tan^{-1}\left(\left|\dfrac{b}{a}\right|\right)$ QII: $\theta = \pi - \tan^{-1}\left(\left|\dfrac{b}{a}\right|\right)$

QIII: $\theta = \pi + \tan^{-1}\left(\left|\dfrac{b}{a}\right|\right)$

QIV: $\theta = 2\pi - \tan^{-1}\left(\left|\dfrac{b}{a}\right|\right)$

| 4.1 | 4.2 | 4.3 | 4.4 | ▶RAD AUTO REAL |

The calculator has built in functions for finding both the modulus and the argument. Both are found in the Complex menu, options 4 and 5. Below are examples.

angle$(3-4\cdot i)$ -.927295

$|3-4\cdot i|$ 5

1/2

| 4.2 | 4.3 | 4.4 | 4.5 | ▶RAD AUTO REAL |

Once the argument, θ, and modulous, r, are found, a complex number, z, can be written as:

$$z = r\big(\cos(\theta)+i\cdot\sin(\theta)\big)$$

This will frequently be abbreviated:

$$z = r\cdot cis(\theta)$$

| 4.3 | 4.4 | 4.5 | 4.6 | ▶RAD AUTO REAL |

Find the modulous and argument of the $2-2\sqrt{3}i$ in polar form. Confirm that the polar form is equivalent.

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| 4.4 | 4.5 | 4.6 | 5.1 | ▶RAD AUTO REAL |

We will study many applications in which polar form of complex numbers is useful. For now we will plot complex numbers given in that form.

| 4.5 | 4.6 | 5.1 | 5.2 | RAD AUTO REAL |

	A r	B θ
◆		
1	15	0
2	15	π/2
3	15	π
4	15	3*π/2
5	15	0

A | r

30.06
y

      •
      5
-20   5   20
      •

-30.06