

# Building Concepts: Ratios and Rational Numbers

TEACHER NOTES

## Lesson Overview

In this TI-Nspire™ lesson, students compare the graphs of different collections of ratios and begin to make conjectures about slope and how to compare slopes in terms of differences in rates of change. Students build tables of ratios using pairs of rational number entries and find associated unit rates.



The points associated with collections of equivalent ratios lie on the graph of the same straight line.

## Learning Goals

1. Use and interpret equivalent ratios when the original ratio is a rational number;
2. use and interpret unit rates for ratios given as rational numbers;
3. identify the graph of a collection of equivalent ratios as a ray through the origin;
4. compare the graphs of collections of different equivalent ratios.

## Prerequisite Knowledge

*Ratios and Rational Numbers* is the ninth lesson in a series of lessons that explore the concepts of ratios and proportional reasoning. The lesson builds on students' prior knowledge of plotting points on a coordinate grid by associating a ratio with an ordered pair of values. Prior to working on this lesson students should have completed *Ratio Tables*, *Comparing Ratios*, and *Connecting Ratios to Graphs*. Students should understand:

- the concept of ordered pairs;
- how to plot points on a coordinate grid;
- how to complete a ratio table;
- the concept of rational numbers.

## Vocabulary

- **rational number**: any number that can be written as a fraction  $a/b$ , where  $a$  and  $b$  are integers and  $b \neq 0$ .

## Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

## Lesson Materials

- Compatible TI Technologies:



TI-Nspire CX Handhelds,



TI-Nspire Apps for iPad®,



TI-Nspire Software

- Ratios and Rational Numbers\_Student.pdf
- Ratios and Rational Numbers \_Student.doc
- Ratios and Rational Numbers.tns
- Ratios and Rational Numbers\_Teacher Notes
- To download the TI-Nspire lesson (TNS file) and Student Activity sheet, go to <http://education.ti.com/go/buildingconcepts>.

### Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:



**Class Discussion:** Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.



**Student Activity:** Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.



**Additional Discussion:** These questions are provided for additional student practice, and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

### Mathematical Background

In this TI-Nspire™ lesson, students compare the graphs of different collections of ratios and begin to make conjectures about slope and how to compare slopes in terms of differences in rates of change. Building Concepts: Ratios and Proportional Relationships, lessons 3 through 8 primarily focused on situations where the initial ratios were whole numbers. In lesson 3, *Building a Table of Ratios*, students created table of ratios that involved whole number entries. In this lesson, *Ratios and Rational Numbers*, students build tables of ratios using pairs of rational number entries and find associated unit rates. *Ratios and Rational Numbers* also builds on the concept developed in lesson 8, *Connecting Ratios to Graphs*, where the points associated with collections of equivalent ratios lie on the graph of the same straight line.

## Part 1, Page 1.3

Focus: This part revisits ratio tables and graphing ratios using rational numbers.

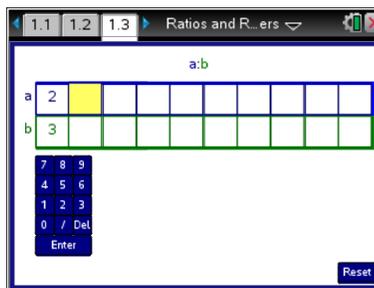
Page 1.3 displays a ratio table.

Set the ratio in the table by highlighting a cell, entering a new value, then pressing **enter**.

Highlight another cell in the top or bottom row in the table and enter a value. The corresponding value in the ratio pair will display.

Delete the contents of a cell by pressing **Del** button or the **del** key on the handheld.

Use the arrow keys to navigate to different cells in the table.



### TI-Nspire Technology Tips

**Reset** returns to the original screen or press **ctrl del** to reset.

**Teacher Tip:** Note that when 0 is entered into one of the cells, the message “Not Allowed” appears because 0:0 is not a ratio. Any line that contains the graph of a collection of equivalent ratios goes through the point (0, 0); however, the point (0, 0) is not a member of the collection.

In Part 1, students revisit ratio tables, but this time the original ratios can be positive rational numbers. Unit rates are suggested in some of the questions as good strategies for solving problems. Students should remember that adding two columns in a ratio table produces another ratio equivalent to those added. The questions below provide good practice in working with fractions.

### Class Discussion

Have students...

Enter  $\frac{1}{2}$  in the highlighted cell.

- **Make a prediction about the corresponding value in the second row of the table. Select **enter** to check your prediction.**
- **If both values in a ratio are multiplied by the same positive number, the ratios are equivalent. Find the number that shows the entries in the second column are equivalent to the ratio 2:3. Explain how you found your answer.**

Look for/Listen for...

Answer:  $\frac{3}{4}$

Answer: You need a number so that the product of the number and 2 is  $\frac{1}{2}$ , or  $2x = \frac{1}{2}$ .

Dividing both sides of the equation by 2 gives  $x = \frac{1}{4}$ . Thus, the number is  $\frac{1}{4}$  because  $\frac{1}{4} \times 2 =$

$\frac{1}{4} \times 3$  is the same as  $\frac{1}{2} : \frac{3}{4}$ .

## Class Discussion (continued)

Have students...

- **Enter  $\frac{1}{2}$  in the second row of the third column. Explain why the values in the third column are also equivalent to the ratio 2:3.**

Reset the TNS lesson and use it to help you answer the following questions.

**A recipe calls for  $\frac{3}{4}$  cup of flour for every  $\frac{1}{2}$  stick of butter.**

- **Find a whole number ratio equivalent to the ratio in the recipe. Explain your reasoning.**
- **How much of each would you need for 6 batches of the recipe?**
- **What is the unit rate of flour per 1 stick of butter? Explain how you can find the answer, and then check using the TNS lesson.**

Look for/Listen for...

Answer: multiplying both values in the ratio by  $\frac{1}{6}$  produces the ratio  $\frac{1}{3} : \frac{1}{2}$ .

Answer: 3:2, 6:4, or any other whole number multiple of 3 and 2. The denominators of each fraction in the ratios are 4 and 2, so using a common denominator as a multiplier for the values in the ratios will replace the fractions with whole numbers, i.e.,  $4 \times \frac{3}{4} = 3$  and  $4 \times \frac{1}{2} = 2$ .

Answer: Multiply both values by 6 to get  $\frac{9}{2}$  or  $4\frac{1}{2}$  cups of flour to 3 sticks of butter.

Answer: You could multiply both values by 2 because you know you need a 1 in a unit rate instead of  $\frac{1}{2}$ , which would make  $\frac{3}{2}$  to 1.

Therefore, the recipe calls for  $1\frac{1}{2}$  cups of flour per 1 stick of butter. To see this using the TNS lesson, enter 1 in the second row of any column.



## Class Discussion (continued)

Have students...

Look for/Listen for...

*TJ walks  $\frac{1}{2}$  mile every  $\frac{1}{4}$  hour. Which of the following strategies can you use to find how many miles he walks per hour? Explain why each is or is not a good method. Check your answer using the TNS lesson.*

- a. *Multiply both values by 4.*
- b. *Divide both values by  $\frac{1}{4}$ .*
- c. *Multiply both values by 2.*
- d. *Divide both values by 2.*
- e. *Reason that if he walked  $\frac{1}{2}$  mile every  $\frac{1}{4}$  of an hour, how far would he go in  $\frac{1}{2}$  hour, then in  $\frac{3}{4}$  hour, and then in 1 hour?*

Answer: Either a, b, or e will give you the right answer, 2 miles per hour. Explanations will vary.



## Student Activity Questions—Activity 1

1. A recipe for salad dressing calls for 2 tablespoons of vinegar and 3 tablespoons of olive oil.
  - a. Using the ratio 2:3 on page 1.3, enter the numbers 1 to 9 in the top row and find the corresponding values in the second row. What do the numbers in the column starting with 7 mean in terms of the vinegar and olive oil?

Answer: 7 tablespoons of vinegar for  $\frac{21}{2}$  or  $10\frac{1}{2}$  tablespoons of olive oil.

- b. Jenny claims she can use the columns for 3 tablespoons of vinegar, 1 tablespoon of vinegar, and 7 tablespoons of vinegar to find the number of tablespoons of olive oil for 11 tablespoons of vinegar. Do you agree with Jenny? Why or why not?

Answer: I agree because if you add equivalent ratios, you get an equivalent ratio. So you could add the  $7 + 1 + 3$  for the vinegar and the corresponding values  $\frac{3}{2} + \frac{9}{2} + \frac{21}{2}$  to get the ratio 11

tablespoons of vinegar to  $\frac{33}{2}$  or  $16\frac{1}{2}$  tablespoons of olive oil.



### Student Activity Questions—Activity 1 (continued)

- c. Use the table in at least two ways to help you figure out how much olive oil you would need for  $5\frac{2}{3}$  cup of vinegar. Describe each of the ways.

Answer: You can set the vinegar at 5 and again at  $\frac{2}{3}$ , and then add the corresponding values for olive oil.  $\frac{15}{2} + 1$ , which is  $7\frac{1}{2} + 1$  or  $8\frac{1}{2}$  tablespoons. Another way would be to write  $5\frac{2}{3}$  as  $\frac{17}{3}$ .

### Part 2, Page 2.2

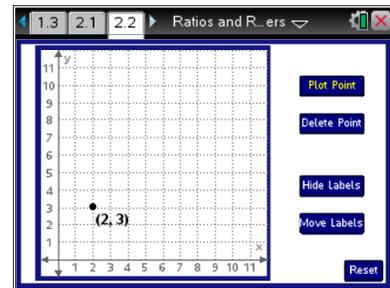
Focus: Graphing equivalent ratios.

On page 2.2 the black dot on the grid can be moved using the arrows on touchpad on the handheld.

Plot points by dragging the points to a coordinate on the screen and pressing **Plot Point**, or by using the arrow buttons to move the point into place, then pressing **enter**.

You can hide the coordinate labels by pressing **Hide Labels** or use the **tab** key to toggle through the options.

You can press the **menu** button to select options to plot, delete, or hide points.



**Teacher Tip:** Have students name some points that lie along the ray. Guide students in a discussion of to remind them that the points are on the same ray because they are associated with a collection of equivalent ratios.



### Student Activity Questions—Activity 2

1. The table below shows the different mixtures of paint that the students made.

	A	B	C	D	E
Yellow	1 can	2 cans	3 cans	4 cans	6 cans
Blue	2 cans	3 cans	6 cans	6 cans	9 cans

[Adapted from Illustrative Mathematics, Grade 7, Art Class Variation 1; [www.illustrativemathematics.org/illustrations/100](http://www.illustrativemathematics.org/illustrations/100)]

- a. How many different shades of paint did the students make? Explain how you can tell.

Answer: There were two different batches because there are two different ratios, one 1:2 and the other 2:3.



## Student Activity Questions—Activity 2 (continued)

- b. **Plot the points (yellow, blue) for the mixtures. What do you notice about the points? Select the points and draw the segment to check your prediction.**

Answer: The points lie on two different lines, each going through the origin.

- c. **Some of the shades of paint were bluer than others. Which mixture(s) were bluer? Show your work or explain how you know.**

Answer: When there were 6 cans of blue paint, one mixture had 3 cans of yellow and the other had 4 cans of yellow, so the first and third mixtures are bluer.

- d. **How does the graph help you think about which of the mixtures is the bluer?**

Possible answer: The graph of points representing the mixtures equivalent to 1:2 is above the graph of the points representing the mixtures equivalent to 2:3. You can also see from the graph that for every 1 can of yellow paint there is about  $\frac{3}{2}$  can of blue paint for the mixtures equivalent to 2:3, while for the mixtures equivalent to 1:2, there is 1 can of yellow for 2 cans of blue.

2. **Three students are reading the same book for English class. On average, Jenn can read 5 pages in 3 minutes, Silvia can read 11 pages in 4 minutes, and Loren can read 3 pages in  $3\frac{1}{2}$  minutes. Use the TNS lesson to graph (number of pages, minutes).**

- a. **Explain how to use the graph to estimate the time it would take each of the students to read 4 pages.**

Answer: You can look at the horizontal axis for 4 pages and draw an imaginary vertical line through the point (4, 0). Finding the corresponding time for the points (4,  $t$ ), where  $t$  represents time in minutes, on each of the three lines shows that Silvia takes about  $1\frac{1}{2}$  minutes, Jenn  $2\frac{1}{2}$  minutes, and Loren almost 5 minutes to read 4 pages.

- b. **Who will read the most number of pages in 15 minutes? How many pages will that person read? Explain your thinking.**

Answer: Using the estimates from part a, Silvia will read the most pages in 15 minutes. Possible explanation: She reads 4 pages in  $1\frac{1}{2}$  minutes so she reads 8 pages in 3 minutes. So multiplying by 5, she would read 40 pages in 15 minutes.



## Student Activity Questions—Activity 2 (continued)

- c. **How could you use the graph to find how many pages each student could read in 1 minute?**

Answer: Think about a horizontal line parallel to the horizontal axis going through time equals 1 on the vertical axis. The points where this line intersects each of the lines representing the equivalent ratios pages: time for each student will give an estimate of the number of pages each student can read in 1 minute. Silvia reads almost 3 pages per minute, Jenn reads a bit more than  $1\frac{1}{2}$  page per minute ( $1\frac{2}{3}$  pages), and Loren reads a little less than 1 page per minute.

3. **The science class built some solar-powered robots and raced them in the school parking lot. The table shows the distance,  $d$ , in meters each of three robots traveled after time,  $t$ .**

	Robot A	Robot B	Robot C
Time (seconds)	1	6	5
Distance (meters)	5	9	2

(Adapted from Illustrative Mathematics, Grade 7, Robot Races, [www.illustrativemathematics.org/illustrations/181](http://www.illustrativemathematics.org/illustrations/181))

- a. **If each robot traveled at a constant speed, find other values for the time and distance for each robot. Plot the points (time, distance) and display the lines for each robot.**

Possible answer: Robot A: 2 seconds for 10 meters; Robot B: 2 seconds for 3 meters; Robot C: 10 seconds for 4 meters.

- b. **Use the graph to find which of the three robots was moving the fastest. Explain your thinking.**

Possible answer: The graph for Robot A is the steepest. In 2 seconds, Robot A went 10 meters, Robot B went 3 meters, and Robot C less than 1 meter, so Robot A went the fastest.

- c. **Suppose there is a Robot D. Find a ratio of time to distance for Robot D that would make Robot D faster than Robot B but not as fast as Robot A. Explain your reasoning. Then, check your work using the TNS lesson.**

Possible answer: Robot D could go anywhere between 3 and 10 meters in 2 seconds.

 Additional Discussion

For each question, explain how you found your answer, and then check your work using the lesson.

Enter the ratios (as fractions not mixed numbers) in the first column on the TNS page.

A recipe for homemade cola uses  $1\frac{1}{4}$  ml of cinnamon oil for every  $\frac{1}{4}$  ml of lime oil in every batch.

- **What is the amount of lime oil for every ml of cinnamon oil?**

Answer:  $\frac{1}{5}$  ml of lime oil. Possible explanation: to make  $\frac{5}{4}$  into 1, you need to multiply by  $\frac{4}{5}$ ; so multiply  $\frac{1}{4}$  by  $\frac{4}{5}$  and you get  $\frac{1}{5}$ .

- **What is the amount of cinnamon oil for every ml of lime oil?**

Answer: 5 ml of cinnamon oil. Possible explanation: to make  $\frac{1}{4}$  into 1, multiply by 4, so  $\frac{5}{4} \times 4 = 5$ .

- **How much of each do you need if you wanted to make 5 batches of cola?**

Answer:  $\frac{25}{4}$  or  $6\frac{1}{4}$  ml of cinnamon oil and  $\frac{5}{4}$  ml of lime oil. Possible explanation: multiply both of the original values by 5.

The times and distances for three more of the solar-powered robots are in the table below. Find another ratio for each robot, then graph (time, distance) for the three robots.

	Robot E	Robot F	Robot G
Time (seconds)	1	5	2.5
Distance (meters)	5	2	3

(Adapted from Illustrative Mathematics, Grade 7, Robot Races Assessment Version, <https://www.illustrativemathematics.org/illustrations/1178>)

- **Which of the following statements about Robot G are true? (Select all that apply.)**

i) Robot G traveled in a different direction than the other two robots.

ii) Robot G traveled 7.5 meters in 9 seconds.

iii) Robot G traveled 1 meter in 2 seconds.

iv) Robot G traveled  $1\frac{1}{5}$  meters per second.

v) Robot G traveled  $\frac{3}{2}$  meters per second.

Answer: ii and iv are true.



## Additional Discussion (continued)

- *How do the speeds of the robots compare?*
  - i) *The Robots all traveled at the same speed but left at different times.*
  - ii) *Robot E is the fastest and Robot F is the slowest.*
  - iii) *There is not enough information given to compare how fast the robots traveled.*

Answer: ii is true.



## Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS lesson.

1. Mark and Steve bike at the same rate. If it takes Steve  $\frac{1}{4}$  hour to bike 6 miles, how long will it take

Mark to bike 8 miles? **Answer:**  $\frac{1}{3}$  hour

2. The ratio of soy sauce to olive oil in a salad dressing is  $\frac{1}{10}$  ml to  $\frac{3}{5}$  ml. How much soy sauce do you

need for 1 ml of olive oil? **Answer:**  $\frac{1}{6}$  ml

The graph below shows the number of pounds of peanuts and the cost in dollars at three different stores.



3. Which of the following statements are true?
- Store C has the highest cost per pound of peanuts.
  - Store C has the lowest cost per pound of peanuts.
  - Store A costs twice as much per pound as store C.
  - Store B only sells peanuts in 2 and 4 pound packages.

**Answer:** b) Store C has the lowest cost per pound of peanuts.



## Sample Assessment Items (continued)

4. What is the price per pound of peanuts at Store A? **Answer: \$1.50 per pound.**
5. On one trip, Helen rode  $\frac{1}{2}$  km in the first 2 minutes and then  $\frac{3}{4}$  km in the next 3 minutes. Which one of the following statements is correct?
  - a. Helen's average speed was greater in the first 2 minutes than in the next 3 minutes.
  - b. Helen's average speed was the same in the first 2 minutes and in the next 3 minutes.
  - c. Helen's average speed was less in the first 2 minutes than in the next 3 minutes.
  - d. It is not possible to tell anything about Helen's average speed from the information given.

(PISA, 2013)

**Answer: b) Helen's average speed was the same in the first 2 minutes and in the next 3 minutes.**



## Student Activity Solutions

In these activities you will work together to interpret ratios as rational numbers and graph equivalent ratios. After completing each activity, discuss and/or present your findings to the rest of the class.



### Activity 1 [Page 1.3]

1. A recipe for salad dressing calls for 2 tablespoons of vinegar and 3 tablespoons of olive oil.
  - a. Using the ratio 2:3 on page 1.3, enter the numbers 1 to 9 in the top row and find the corresponding values in the second row. What do the numbers in the column starting with 7 mean in terms of the vinegar and olive oil?

*Answer: 7 tablespoons of vinegar for  $\frac{21}{2}$  or  $10\frac{1}{2}$  tablespoons of olive oil.*

- b. Jenny claims she can use the columns for 3 tablespoons of vinegar, 1 tablespoon of vinegar, and 7 tablespoons of vinegar to find the number of tablespoons of olive oil for 11 tablespoons of vinegar. Do you agree with Jenny? Why or why not?

*Answer: I agree because if you add equivalent ratios, you get an equivalent ratio. So you could add the  $7 + 1 + 3$  for the vinegar and the corresponding values  $\frac{3}{2} + \frac{9}{2} + \frac{21}{2}$  to get the ratio 11 tablespoons of vinegar to  $\frac{33}{2}$  or  $16\frac{1}{2}$  tablespoons of olive oil.*

- c. Use the table in at least two ways to help you figure out how much olive oil you would need for  $5\frac{2}{3}$  cup of vinegar. Describe each of the ways.

*Answer: You can set the vinegar at 5 and again at  $\frac{2}{3}$ , and then add the corresponding values for olive oil.  $\frac{15}{2} + 1$ , which is  $7\frac{1}{2} + 1$  or  $8\frac{1}{2}$  tablespoons. Another way would be to write  $5\frac{2}{3}$  as  $\frac{17}{3}$ .*



### Activity 2 [Page 2.2]

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- a. How many different shades of paint did the students make? Explain how you can tell.

*Answer: There were two different batches because there are two different ratios, 1:2 and 2:3.*

- b. Plot the points (yellow, blue) for the mixtures. What do you notice about the points? Select the points and draw the segment to check your prediction.

*Answer: The points lie on two different lines, each going through the origin.*



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- c. Some of the shades of paint were bluer than others. Which mixture(s) were bluer? Show your work or explain how you know.

*Answer: When there were 6 cans of blue paint, one mixture had 3 cans of yellow and the other had 4 cans of yellow, so the first and third mixtures are bluer.*

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*Possible answer: Robot D could go anywhere between 3 and 10 meters in 2 seconds.*