Mathematical Methods (CAS) 2002 Examination 1 part 1 sample solutions Q 14 to 27

Note: To use *Derive* efficiently, students should be familiar with the 'tick plus equals' and 'tick plus approximately equals' evaluation buttons. These simultaneously 'author' and 'evaluate' expressions exactly and numerically respectively. Students should also be familiar with the use of defined functions in the form f(x):= *rule of the function*, such as in the sample solutions for Question 10.

Some questions are conceptual in nature, that is, technology will not be of assistance, for example, Question 7. For other questions, such as Question 1, the facility of *Derive* to quickly produce scaled graphs, and the like, means that such problems could be tackled by inspection of each alternative, although this is not a recommended approach.In many cases students will reason what is likely to be the answer, and then confirm this with *Derive*.

Question 14

B is the correct answer.This can be identified by application of the chain rule pattern for the composite function log(f(x)). The corresponding derivative is f'(x)/f(x), as f(x) = cos(2x), this means that A and B are the only possibilities. As the derivative of cos(2x) is −2sin(2x) B is correct (the negative is sufficient to discriminate here). Alternatively,differentiation by *Derive* gives:

#1: $\mathrm{LOG}(\mathrm{COS}(2\cdot x))$

#2: $\frac{d}{dx}\mathrm{LOG}(\mathrm{COS}(2\cdot x))$

#3: $-\ 2\cdot\mathrm{TAN}(2\cdot x)$

Question 15

E can be eliminated since the normal is a linear function. As each of A to D has a different gradient, this information is sufficent to identify the correct alternative.The gradient of the normal is −1/f'(π), where f(x) = x*sin(x). Hence the correct alternative is D, since the gradient is 1/π.

#4: $x\cdot\mathrm{SIN}(x)$

#5: $-\frac{1}{f'(\pi)}$

#6: $$\frac{1}{\pi}$$

alternatively, the rule of the normal, and hence the correct alternative, can be obtained in a single evaluation using the form y – f(π) = –1/f′(π)(x–π) and solving for y:

#7: $$\text{SOLVE}\left(y - f(\pi) = -\frac{1}{f'(\pi)}\cdot(x - \pi),\ y\right)$$

#8: $$y = \frac{x - \pi}{\pi}$$

Question 16

This can be evaluated directly using a defined function, to identify C as the correct alternative:

#9: $f(x) := e^{-x} - 1$

#10: $f'(0)$

#11: $$-1$$

Question 17

This is a formulation question, no evaluation is required. With x = 3, h = 0.02, so by substitution (without evaluation) the correct form is A.

Question 18

A conceptual question. All graphs include required zeroes of the function, C can be eliminated as it does not have the correct location for stationary points.As the derivative is required to be positive for x < –1, only B and D are possibilities. B is the only alternative for which the derivative is negative (except at x = 0) for x > –1.

Question 19

This question tests knowledge of the chain rule, the derivative of g(x) = e^(f(x)) is g′(x) = f′(x)*e^(f(x). since g′(x) = –2xe^(–x^2)) this matches f(x) with –x^2. Thus the correct alternative is A. Another approach is to anti-differentiate g′(x) and hence identify f(x) = –x^2:

#12: $$-2\cdot x\cdot e^{-x^2}$$

#13: $$\int -2\cdot x\cdot e^{-x^2}\,dx$$

#14: $$e^{-x^2}$$

Question 20

The correct alternative, B, can be identified by direct evaluation of an anti–derivative, and inclusion of an arbitrary constant c:

#15: $2\cdot \mathrm{COS}(5\cdot x)$

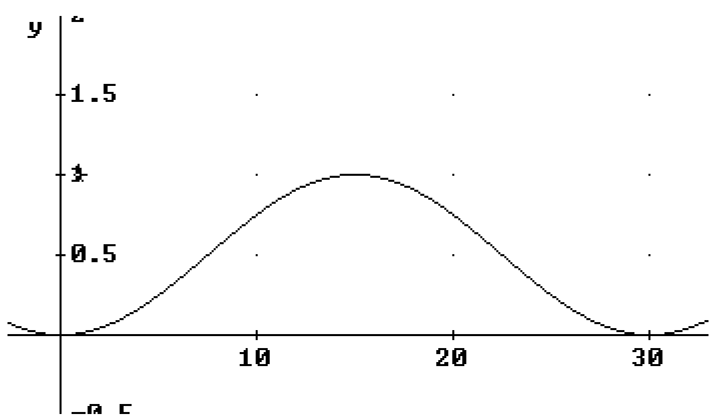
#16: $\int 2\cdot \mathrm{COS}(5\cdot x)\,dx$

#17: $$\frac{2\cdot \mathrm{SIN}(5\cdot x)}{5}$$

Question 21

This is a formulation and evaluation question, where distance travelled is modelled by a definite integral. As there are no turning points (ie where v(t) = 0)during the time interval specified, and the function is non–negative over this interval (see graph), the definite integral can be used directly without having to partition the interval or use modulus, to identify D as the correct alternative:

#18: $\mathrm{SIN}\left(\frac{\pi\cdot t}{30}\right)^2$



#19: $\displaystyle\int_0^{30} \text{SIN}\left(\frac{\pi\cdot t}{30}\right)^2 \, dt$

#20: 15

Question 22

This is a conceptual question, involving recognition that where a function changes sign over an interval, the definite integral does not give the area bounded by the function and the horizontal axis over that interval. The required area is given by the definite integral from a to b, plus the *negative* of the definite integral from b to c. The latter effect can be achieved by changing the order of the terminals in the second definite integral, thus, D is the correct alternative.

Question 23

The anti–differentiation for this question could be carried out by hand, the required definite integral being evaluated as –x^2 + 8x from 3.5 to 4 (students should be able to do this by hand):

#21: $-16 + 32 - - 3.5^2 - 8\cdot 3.5$

#22: 0.25

to obtain the correct answer B. Alternatively, it may be evaluated directly using *Derive*:

#23: $- 2\cdot x + 8$

#24: $\displaystyle\int_{3.5}^{4} (- 2\cdot x + 8) \, dx$

#25: $\dfrac{1}{4}$

Question 24

This is a formulation question, no evaluation is required. Students should recognise that the hypergeometric distribution applies, and that Pr(at least one chip of value $10) in the sample = 1 – Pr(no chip of value $10) in the sample. Thus the answer is 1 $-{}_{15}C_4/{}_{20}C_4$, that is, alternative A.

Question 25

This question requires the solution of two simultaneous equations in n and p, which can be readily done without any calculation. The mean μ = 10 = np and the variance (standard deviation squared), σ^2 = 9 = np(1−p). Thus 9 = 10(1−p), so p = 0.1, and A is the correct alternative. These can also be solved, for both n and p, using *Derive*:

#26: SOLVE([n·p = 10, n·p·(1 − p) = 9], [n, p])

#27: $$\left[n = 100 \wedge p = \frac{1}{10}\right]$$

Question 26

This question tests understanding of the transformation between the standard normal distribution X~N(0,1), some other normal distributions, X~N(u,σ^2), and the symmetry of a normal distribution. So, if μ = 4.7 and σ = 1.2 then Pr(X < 3.5) =
Pr(Z < −1) since Z = (x − μ)/σ = (3.5 − 4.7)/1.2 = −1. By symmetry of the standard normal distribution, Pr(Z < −1)is the same as Pr(Z > 1), alternative B.

Question 27

The middle 95% will be in the range μ − 6 to μ + 6, thus the upper 97.5 % will be in the range μ − 6 to infinity. For each of the alternatives A − D the largest of these values of μ, u = 254 gives 2.5% of packets less than 254 − 6 = 248, so alternative E is the only one with a large enough value for μ that could work anyway. The problem can also be solved directly using *Derive*:

#28: NSOLVE(NORMAL(250, μ, 3) = 0.01, μ)

#29: $$\mu = 256.9790436$$

thus, as before the correct alternative is E.