EXPLORATIONS

Activity 8

Objective

- To learn to quickly add lists of numbers by adding like terms and using the distributive property

Materials

- TI-73 calculator
- Student Worksheet

Let's Do Summagic

In this activity you will:

- discover the magic (mathematics) in summagic
- use the distributive property

You will need to know this math vocabulary:

- distributive property
- greatest common factor
- coefficient
- like terms

Introduction

Have you ever seen anyone do mental math tricks quickly and wonder how they did it? In this activity, you will learn a trick that will involve adding a list of ten numbers in a split second.

The list is generated in a special way. Do you see a pattern?

2, 4, 6, 10, 16, 26, 42, 68, 110, 178

Problem

Listen as your teacher demonstrates the magic. Write the ten numbers of the class-generated list in Table 1 on the Student Worksheet, then find the sum.

Activity

Part 1

1. On the Home screen, find the sum of the ten numbers in Table 1 on the Student Worksheet and determine if your teacher was correct. (If you are not currently at the Home screen, press [2nd][QUIT] before you begin.)
2. Working with your teacher, complete Table 2 on the Student Worksheet. Use the two numbers 7 and 5 to numerically find the next eight terms.

Part 2

1. Listen as your teacher explains the process for determining the sum of a list.
2. Work with a partner to choose your own numbers and find the sum using the *summagic* rule.
3. Check your sum on the calculator using **sum(L2)**.

Name _____

Date _____

Activity 8

Let's Do Summagic

Record your results on the table below. Then answer the questions about the activity.

Table 1

1	2	3	4	5	6	7	8	9	10

Table 2

L1			L2
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

1. In Table 2 above, how many 7's are there?

_____

2. How many 5's?

_____

3. Write the mathematical expression as: ___ x 7 + ___ x 5

4. What factor do 55 and 88 have in common?

_____

_____

5. Rewrite the expression as: (11)(5)(7) + (11)(8)(5) = 11 (5 x 7 + 8 x 5). Verify that the expression is the same as 55(7) + 88(5). (Use your calculator to find the value of all three mathematical expressions.)

_____

_____

_____

6. Is the expression 5(7) + 8 (5) in our numerically generated list? If so, what number in the list?

_____

_____

Teacher Notes

Activity 8

Let's Do Summagic

Math Strand

- Algebraic reasoning
- Number sense

Materials

- TI-73 calculator
- Student Worksheet (page 71)

Students will discover the rule for the "summagic" problem by adding like terms and using the distributive property. They will use factoring to uncover the magic and explain the mathematics.

Vocabulary

Distributive property	For all real numbers a, b, c, a(b+c)= ab + bc
Greatest common factor	the greatest factor or divisor common to a set of 2 or more numbers
coefficient	the number in front of the variable; for example, in 7m, 7 is the coefficient
like terms	terms that contain the same variable(s) to the same power

Classroom Management

The majority of this activity is teacher-directed. The activity is divided into two parts. Part 1 could be used by itself for younger students. An abbreviated Part 1 along with Part 2 could be used for more experienced students.

Problem

Ask the students to explain how the numbers were generated in the opening paragraph. Draw a table on the board similar to the table in the opening problem. Select a student to randomly choose a one-digit number and write it as the first number in the table. Choose a second student to pick a second number and write it as the second number in the table. Magically, tell the class the sum by mentally multiplying the 7th number by 11. Have the students find the sum of the list of ten numbers on the Home screen to verify your answer.

Multiply by 11 Trick:

1. 32 x 11 = 3 2

3 5 2

2. 109 x 11 = 1 0 9

1 1 9 9

3. 67 x 11 = 6 7

6 13 7

7 3 7

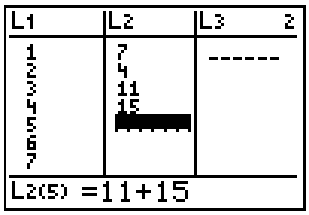
Activity

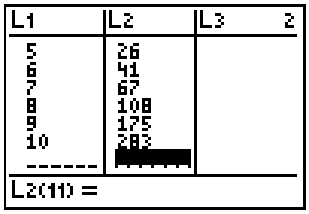
Part 1

1. For instructions on accessing lists, see Appendix A. Ask students to enter the numbers 1-10 as the first ten elements in **L1**. Choose a student to randomly pick a number and enter it as the first element in **L2**. (Any number will work, but you will probably want your students to choose numbers from 1-10.) Choose a second student to randomly pick a second number and enter it as the second element in **L2**.

a. Add the two numbers together and put the sum as element 3 of **L2**. Then add element 3 plus element 2 and put the sum as element 4. You can type the addition problem into an element: that is, to add 11 and 15, type **11 + 15** ENTER and the number **26** will appear as the next number in the list.

b. Continue **L2** by adding the previous two elements until you have filled in the tenth element.





2. You can now magically declare the sum of the ten numbers in **L2**. In the example above, the sum is 737.

a. To find the sum of **L2**, multiply the seventh element in **L2** by 11 quickly. An easy way to multiply by 11 is illustrated above.

b. You say "The sum of this list is _____. Let's check to see if this sum is correct." Go to the Home screen to find the sum of **L2**. Press 2nd [STAT] ▶ ▶ and select **7:sum(** 2nd [STAT] choose **L2**) ENTER. Record the sum and verify that the sum is correct.

3. Ask, "How was I able to find the sum so quickly?"

a. Give students a chance to explore **L2** and make conjectures. "Do you think I could do it again?"

b. Have two other students choose two new numbers and repeat the process.

c. After you have stunned your class by correctly finding the sum, develop the following on Table 2 to investigate the teacher's magic. Have the students complete Table 2 along with you. Use the two numbers 7 and 5 to numerically find the next eight terms. Ask, "What pattern can you see? Can we write these in a briefer form?" Help the students find the next two columns.

d. After the table is completed, have the students answer the questions following the table.

L1			L2
1	7	=7	=7
2	5	=5	=5
3	7+5	=7 + 5	=12
4	(7+5) + 5	=7 + 2(5)	=17
5	(7+5+5) + (7+5)	=2(7) + 3(5)	=29
6	(7+5+5+7+5) + (7+5+5)	=3(7) + 5(5)	=46
7	etc.	=5(7) + 8(5)	=75
8		=8(7) + 13(5)	=121
9		=13(7) + 21(5)	=196
10		=21(7) + 34 (5)	=317

Answers to Student Worksheet

1. 55

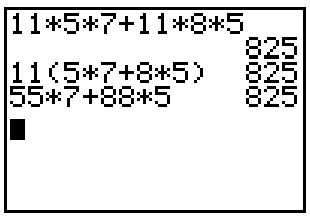
2. 88

3. $\underline{55} \times 7 + \underline{88} \times 5$

4. 11

5. See figure at the right.

6. Yes, the 7^{th} number.



Extension

1. Develop the following list on the board.

1	a	=a
2	b	=b
3	b+a	=b+a
4	(b+a)+b	=a+2b
5	(b+a+b)+(b+a)	=2a+3b
6	(b+a+b+b+a)+(b+a+b)	=3a+5b
7	etc.	
8		
9		
10		

2. Ask the students:

- *How can we find the sum of list two in terms of a and b?*
- *How many a's are there? How many b's are there?*

Find the sum of all ten elements in **L2** by combining the number of a's and number of b's to get the sum. (55a+88b)

3. Ask the students:

- *What do you notice about the numbers in front of the variables?*

Lead the discussion to find that the numbers 55 and 88 are multiples of 11. Ask:

- *How could we write this expression in an equivalent form?*

Using the distributive property the expression can be rewritten as: 55a+88b=11(5a)+11(8b)=11(5a+8b).

4. Ask the students:

- *Do you notice the expression 5a+8b in the list above?* (It is the seventh element in **L2**.)
- *How does the sum of the list compare to this term?* (It is eleven times the term.)

Therefore, if you find the seventh element and multiply it by eleven you will have the sum of the list.

- *Will this work for other numbers?*

5. Have the students work in pairs to choose their own 2 numbers and repeat the activity. Ask them to find the sum using the summagic rule. Have them go to the Home screen and check their sum using sum (**L2**). Check with each of the pairs to see if they were able to find the correct sum.

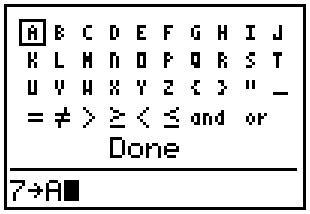
6. To wrap up:

   a. Have a few of the groups explain to the class how they found their sum.

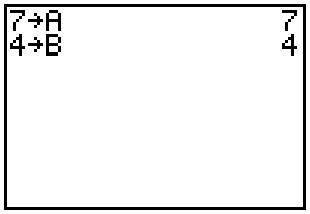
   b. Ask students to write the rule in their own words.

Going Further

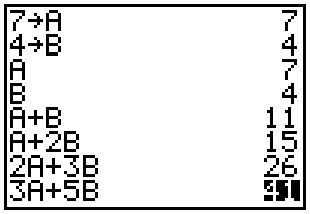
1. Go to the Home screen. Clear the Home screen. Choose two numbers that you have found the sum for previously. For example, use 7 and 4. Let a=7 and b=4. Type [7] [STO▸] and press [2nd][TEXT], highlight **A**, [ENTER], use cursor to select **Done**, [ENTER]. Follow the same procedure to store 4 to **B**.

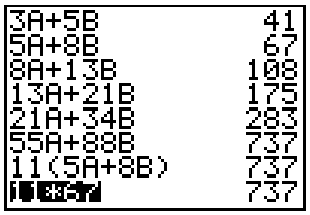


2. Using **A** and **B** on the Home screen, find the terms of the list as shown at the right.



3. Discuss each line and compare the lines to the list of numbers previously generated on the board. Show that each algebraic expression gives the correct numerical value. Enter the sum as an algebraic expression **55A+88B**, then in the factored form of **11(5A+8B)**, (which is really 11 times the seventh term). Both expressions will display the same numerical value. Finally, multiply eleven by the seventh term demonstrating the "Summagic" rule.





4. Have the students amaze a family member by finding the sum using the "Summagic" rule. Have them bring back a written note from the family member (or other suitable person) explaining how anyone, not knowing anything about how it works, could find the sum of the ten terms.

5. A famous mathematician, Leonardo de Pisa, better known as Fibonacci, investigated a sequence of numbers now known as the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 35, …. Each number is the sum of the two previous numbers. This sequence is found in many places in nature, including pine cones, sunflowers, ridges on certain seashells. Also, when each number in the series is divided by the previous term, the results get

closer and closer to a value known as the golden ratio. Notice that the coefficients of **a** and **b** in the summagic rule are the numbers in the *Fibonacci* sequence. Have students research Fibonacci and the Fibonacci sequence and report their findings.