Case File 4

Flipping Coins: Density as a characteristic property

Expose a counterfeiter by proving his "old" coins have a "new" density.

March 11

Times Standard

A Case of Coinery

Counterfeiting ring cracked

NEW THETFORDSHIRE, March 10: Coin collector Clark Esposito thought it was his lucky day when a stranger entered his shop with a plastic sleeve full of rare, mint 1877 indian head pennies. The seller, Zeus Duncan, said he had kept the coins in a locked safe since they were given to him by his father 20 years ago. However, Mr. Esposito's lucky day soon turned into a lucky break for police investigating a counterfeiting ring operating in the city.

"As soon as I picked up the sleeve, I knew something was wrong," said Mr. Esposito. "It was far too light to contain so many pennies." Fearing he was the target of a counterfeit operation, Mr. Esposito called the police, who arrived and took Mr. Duncan into custody. Police later proved that the coins were counterfeit. Instead of being genuine 1877 pennies, they were found to be modern pennies that had been re-stamped.

"This was the work of a master counterfeiter," says chief investigator Molly Harbert. "The 1877 indian head cent, when in good or mint condition, can sell for tens of thousands of dollars."

A real 1877 indian head cent (left) and one of the counterfeit pennies (right) are identical in size and relief.

Forensics Objective

- identify counterfeit coins based on the characteristic property of density

Science and Mathematics Objectives

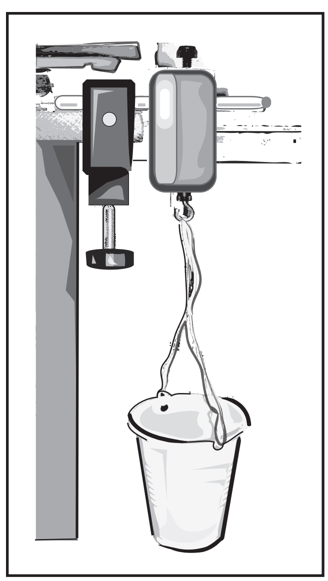
- model data using a linear equation
- interpret the slope and intercept values from a linear model
- identify a characteristic property of a substance

Materials (for each group)

- TI-83/TI-84 Plus™Family
- Vernier EasyData™ application
- Vernier EasyLink™
- Dual-Range Force Sensor
- clamp or heavy tape
- light plastic coffee cup
- string
- 20 pennies dated 1963–1981
- 20 pennies dated 1982
- 20 pennies dated after 1982

Procedure

1. Set up your materials.
   a) Use a pencil to poke small holes on opposite sides of the coffee cup, near the top rim. Thread a piece of string through the holes, and then tie the ends of the string together to form a loop to hang the cup from.
   b) Separate each group of 20 pennies into four stacks of 5 pennies each. As you do this, confirm that you have 20 pre-1982 pennies, 20 pennies dated 1982, and 20 post-1982 pennies.
   c) Set the range switch on the Dual-Range Force Sensor to ±10 N.
   d) Secure the force sensor to the edge of a table. The sensor must be positioned with the hook closest to the ground and should remain level at all times. The diagram below gives an example of one way to set up the force sensor.



2. Plug the Dual-Range Force Sensor into EasyLink, and plug EasyLink into the USB port on the calculator. The calculator should automatically turn on and take you to the force sensor Main screen.

> At the bottom of the Main screen are five options (File, Setup, Start, Graph, and Quit). Each of these options can be selected by pressing the calculator key located below it (Y=, WINDOW, ZOOM, TRACE, or GRAPH).
>
> TIP!

3. Notice that the mode on the bottom of your screen is **Time Graph**. You need to change this to **Events with Entry** so that you can tell the calculator when to record the force.
   a) Select Setup from the Main screen.
   b) Select option **3: Events with Entry**. Your Main screen should now read **Events with Entry** on the bottom.

4. In this experiment, you want to measure the weight of the pennies only, not the pennies, cup, and string. To tell the calculator not to record the weight of the cup and string, you need to zero the force sensor.
   a) Hang the empty cup from the hook on the force sensor.
   b) Select Setup and select option **7: Zero**.
   c) Wait until the reading is stable (make sure the cup is *not* moving). Select Zero (not 0) to zero the force sensor. This will set the current weight reading to 0, so the sensor will ignore the weight of the cup and the string.

5. You are now ready to record the weights of different numbers of pennies.
   a) The empty cup should be hanging from the hook on the force sensor. Select Start to begin data collection.
   b) When the weight reading of the empty cup is stable (it should be very close to 0), select Keep.
   c) The calculator will then ask you to enter a value. Type **0** for the number of pennies now in the cup. Select OK to store this weight-number data pair.
   d) Place five of the pre-1982 pennies in the cup, and wait until the reading is stable. Select Keep.
   e) Type **5** as the value for the number of pennies in the cup. Select OK to store this weight-number data pair.
   f) Continue with this procedure, using increasing numbers of pre-1982 pennies. Each time you keep a data pair, enter the *total* number of pennies in the cup as the number value.
   g) Select Stop when you have finished collecting data for the pre-1982 pennies.

6. The calculator screen will now show a graph with number of pennies on the *x*-axis and weight, in newtons, on the *y*-axis. The graph should appear linear (a straight line). If it does not, select Main to return to the Main screen, and then repeat steps 4 and 5.

7. Determine the equation that describes the relationship between weight and number of pennies.
   a) Select Anlyz.
   b) Select option **2: Linear Fit**. This will tell the calculator to compute the equation of the straight line that best fits your data.
   c) The calculator will display values for **a**, **b**, and **R**. Record these values in the Evidence Record.
   d) The equation of a straight line is $y = ax + b$. Using the values shown on the screen, write the equation that best fits your data into your Evidence Record. (For example, if **a** = 3 and **b** = 5, the equation for the line is $y = 3x + 5$.)
   e) Select OK to plot a line through the data points.
   f) Select Main to return to the Main screen.

8. Repeat steps 4–7 for the pennies dated 1982.

9. Repeat steps 4–7 for the pennies dated after 1982.

NAME: ____

DATE: ____

Evidence Record

Penny Date	Equation of the Best Fit Line: $y = ax + b$
1963–1981	a: ____________ b: ____________ R: ____________ Equation ($y = ax + b$): ____________
1982	a: ____________ b: ____________ R: ____________ Equation (y = ax + b): ____________
After 1982	a: ____________ b: ____________ R: ____________ Equation (y = ax + b): ____________

Case Analysis

1. The equation for a straight line is $y = ax + b$, where x and y are coordinates on the line, a is the slope of the line, and b is the y-intercept (the value of y when $x = 0$).
   In this case, y is the weight of the pennies, in newtons, x is the number of pennies, and a is the "density" of the pennies. What are the units of "density" in this equation?
2. Explain why the *y*-intercept, b, should be 0.
3. Was the value of b that you recorded for each group of pennies equal to 0? If not, explain why not.
4. How do the "densities" of the three sets of pennies compare? Based on your measurements, what do you think probably happened to the composition of the penny in 1982?
5. Use the appropriate equation to determine the weight of 5000 pennies from 1980. Show the equation you used and how you rearranged and/or substituted into the equation. Underline your answer.
6. Use the appropriate equation to determine the weight of 5000 pennies from 2005. Show the equation you used and how you rearranged and/or substituted into the equation. Underline your answer.
7. In this activity, "density" is in quotation marks because the slope of the line, a, is not actually density; a is just a *measure* of density. Explain why the value of a is not really density.

8. Why can you still use slope, a, as a measure of density?
9. What could have made the penny "densities" you calculated inaccurate?
10. From 1864 to 1962, pennies were made of 95% copper and 5% zinc-tin alloy. From 1962 to 1981, pennies were made of 95% copper and 5% zinc. Since 1983, pennies have been made of 97.5% *zinc* and 2.5% *copper*. Zinc is significantly less dense than copper. Tin is slightly more dense than zinc but still much less dense than copper.
    If the suspect's coins are genuine 1877 pennies, how should their density compare with the densities of the pennies you measured in this activity?
11. Police measured the weight of five of the suspect's coins and found them to be 0.06 N (about 0.22 oz) each. Based on the data you collected, explain how the police knew that the suspect's coins were fakes. (Hint: What is the weight of five pennies from after 1982?)
12. R is a measure of how well the line fits the data points. A large value of R indicates that the line is a good fit to the data points. Which group of pennies showed the best fit to a straight line? How do you know?

Case File 4
Flipping Coins: Density as a characteristic property

Teacher Notes

Teaching time: one class period

This lab introduces the concept of density and uses it to distinguish between pennies minted in different years.

Tips

- Before assigning this activity, you may want to review the concept of density, the formula for density (mass divided by volume), and the equation of a line ($y = ax+b$).
- If pennies are scarce, have groups share batches.
- It is important that the force sensor be firmly attached to a stationary object; it will probably not give good results if you have a student hold it. You may need to experiment a bit to devise a mechanism to hold the force sensor in place on the tables available in your classroom; the image given in the procedure is just a suggestion.

Background Information

As shown in the table below, the U.S. Department of the Treasury has changed the composition, and thus the density, of the penny several times.

Date	Penny Composition
1793–1837	Pure copper
1837–1857	95% copper, 5% tin and zinc
1857–1864	88% copper, 12% nickel
1864–1942	95% copper, 5% tin and zinc
1943	Zinc-coated steel; pure copper in a few
1944–1962	95% copper, 5% tin and zinc
1962–1982	95% copper, 5% zinc
1982–present	97.5% zinc, 2.5% copper (copper-coated zinc)

Source: http://www.usmint.gov/about_the_mint/fun_facts/index.cfm?flash=yes&action=fun_facts2

Metal	Density (g/cm^3)
Copper	8.92
Nickel	8.91
Tin	7.31
Zinc	7.14

The general equation for the density of a coin is the following:

density = [(percentage of metal A) × (density of metal A)]
+ *[(percentage of metal B) × (density of metal B)]*

Density of pennies from 1963 to 1981 =
$(0.95)\ (8.92\ g/cm^3) + (0.05)\ (7.14\ g/cm^3) = 8.83\ g/cm^3$

Density of pennies since 1983 =
$(0.975)\ (7.14\ g/cm^3) + (0.025)\ (8.92\ g/cm^3) = 7.18\ g/cm^3$

The composition was changed in 1982 because the cost of making the penny was more than the penny was worth!

Modifications

- If you have trouble finding enough 1982 pennies, students can skip step 8.
- If students are having trouble understanding how to interpret the equations that the calculator generates, you may want to have them sketch and label the graphs in step 6.
- Advanced students can use the chemical composition and density data in the Background Information to calculate and compare the densities of pennies minted in different years and then compare those densities to their experimental data. Depending on how advanced the students are, you may have them research the penny compositions and metal density information themselves. It is important that they understand that the slope of the weight–number-of-pennies line is *not* actually density (see Case Analysis questions 7 and 8).
- Advanced students can try to convert the units of a (newtons per penny) to the standard density units (g/cm^3). Students can also try to determine the volume of a penny (they will need to convert weight to mass and use the calculated density of a penny).

Sample Data

Penny Date	Equation of the Best Fit Line: $y = ax + b$
1963–1981	a = 0.02969694 b = -0.0095746 R = 0.99792326 Equation ($y = ax + b$): $y = 0.02969694x + (-0.0095746)$
1982	a = 0.02384556 b = -0.0105673 R = 0.99629841 Equation ($y = ax + b$): $y = 0.02384556x + (-0.0105673)$
After 1982	a = 0.01862232 b = -0.0332276 R = 0.99313258 Equation ($y = ax + b$): $y = 0.01862232x + (-0.0332276)$

Case Analysis Answers

1. The equation for a straight line is $y = ax + b$, where x and y are coordinates on the line, a is the slope of the line, and b is the y-intercept (the value of y when $x = 0$).
   In this case, y is the weight of the pennies, in newtons, x is the number of pennies, and a is the "density" of the pennies. What are the units of "density" in this equation?
   These units are newtons per penny.
2. Explain why the y-intercept, b, should be 0.
   When x = 0, there are no pennies being measured. The weight of no pennies should be 0.
3. Was the value of b that you recorded for each group of pennies equal to 0? If not, explain why not.
   Answers will vary. Possible reasons for non-zero b values include air currents in the room or movement of the force sensor. It is difficult to collect the first point exactly when the force sensor measures 0.
4. How do the "densities" of the three sets of pennies compare? Based on your measurements, what do you think probably happened to the composition of the penny in 1982?
   The pennies minted before 1982 are denser than the pennies minted after 1982. The density of pennies minted in 1982 falls between the other two groups. The composition of the penny was probably changed in 1982 in a way that made it less dense.
5. Use the appropriate equation to determine the weight of 5000 pennies from 1980. Show the equation you used and how you rearranged and/or substituted into the equation. Underline your answer.

$$y = 0.02969694\,x - 0.0095746$$

$$y = 0.02969694\,(5000) - 0.0095746 = \underline{148\text{ N}}$$

6. Use the appropriate equation to determine the weight of 5000 pennies from 2005. Show the equation you used and how you rearranged and/or substituted into the equation. Underline your answer.

$$\boldsymbol{y = 0.01862232\,x - 0.0332276}$$

$$\boldsymbol{y = 0.01862232\,(5000) - 0.0332276 = \underline{93\ N}}$$

7. In this case, "density" is in quotation marks because the slope of the line, a, is not actually density; a is just a *measure* of density. Explain why the value of a is not really density.
   Density is mass divided by volume. The a here equals weight (force, in newtons) divided by number of pennies. Although a penny is essentially a unit of volume, weight is not the same thing as mass.
8. Why can you use a here as a measure of density?
   You can use weight divided by number because you can assume that the ratio of weight to mass does not, change and that every penny has the same volume, regardless of the year it was minted. (The ratio of weight to mass does not change because weight is a measure of the force of gravity on a mass. Because every penny is in the same place, the force of gravity is the same on each, so only the mass affects the measured weight.)
9. What can make the penny "densities" you calculate inaccurate?
   The pennies may be worn down and so have less volume. If they are corroded, they may have slightly different compositions. If the force sensor is not kept stable, the readings can be inaccurate.
10. From 1864 to 1962, pennies were made of 95% copper and 5% zinc-tin alloy. From 1962 to 1981, pennies were made of 95% copper and 5% zinc. Since 1983, pennies have been made of 97.5% *zinc* and 2.5% *copper*. Zinc is significantly less dense than copper. Tin is slightly more dense than zinc but still much less dense than copper.
    If the suspect's coins are genuine 1877 pennies, how should their density compare with the densities of the pennies you measured in this activity?
    They should be more dense than any of the pennies in the activity.
11. Police weighed the suspect's coins and found that five of them weighed 0.06 N (about 0.22 oz). Based on the data you collected, explain how the police knew that the suspect's coins were fakes. (Hint: What is the weight of five pennies from after 1982?)
    Five pennies from 1877 should weigh significantly more than 0.06 N, which is the approximate weight of five pennies from 2005. Therefore, the suspect's pennies were fakes.
12. R is a measure of how well the line fits the data points. A large value of R indicates that the line is a good fit to the data points. Which group of pennies showed the best fit to a straight line? How do you know?
    Answers will vary. In the case of the sample data, the 1963–1981 pennies showed the best fit (the largest value of R).