Math Objectives

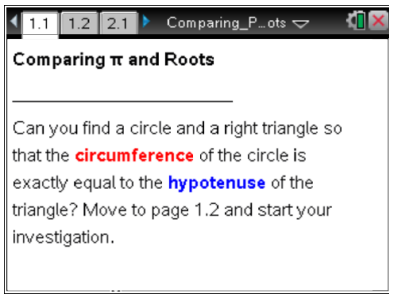
- Students will compare the circumference of a circle with the hypotenuse of a right triangle in order to compare the sizes of irrational numbers.
- Students will use measurements, formulas and calculator computations, and formulas and exact calculations to make comparisons.
- Students will use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2) (CCSS).
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).
- Students will attend to precision (CCSS Mathematical Practice).

Vocabulary

- rational number
- irrational number
- real number
- decimal approximation
- precision

About the Lesson

- This lesson involves manipulating the radius of a circle and the sides of a right triangle in an attempt to set the circumference of the circle equal to the hypotenuse of the right triangle
- As a result, students will:
  - Unwrap the circle alongside the hypotenuse.
  - Use measurements to find decimal approximations of lengths in order to compare them.
  - Use the circumference formula and the Pythagorean theorem to find decimal approximations of the lengths using a calculator.
  - Use the formulas to find exact lengths.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can change the number of displayed digits in different applications by accessing Settings from the Home menu

Lesson Files:

Student Activity

Comparing_Pi_and_Roots_Student.pdf

Comparing_Pi_and_Roots_Student.doc

TI-Nspire document

Comparing_Pi_and_Roots.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

TI-Nspire™ Navigator™ System

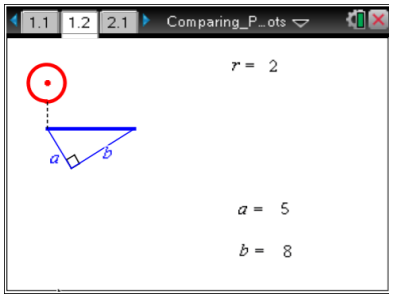
- Send a file.
- Screen Capture.
- Quick Poll.

Live Presenter.

Discussion Points and Possible Answers

Tech Tip: Make sure that students moved the center of the circle all the way in order to fully unwrap the circle. To change the values of r, a, and b, students should move the cursor over the number and double-click while the number is highlighted. They can then type a new number. If students accidentally click on the variable, press esc and move the cursor to the right of the number to highlight the number and not the variable.

Move to page 1.2.



1. Grab and drag the center of the circle to "unwrap" the circle. In order to change the values of a and b, move the cursor over the number, and double-click while the number is highlighted. Then type the new number.
   a. Can you find integer values for a and b for the sides of the right triangle so that the length of the hypotenuse appears to be nearly the same as the length of the circumference of the circle with the radius, $r = 2$? What are some possible values for a and b?

Answer: The possible sets of lengths that students might accept for visual comparison are (4, 12), (6, 11), (9, 9).

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

Teacher Tip: At first, students might have different triangles since they compare the lengths visually and use trial and error to find values of a and b. Discuss the reasons for having different answers.

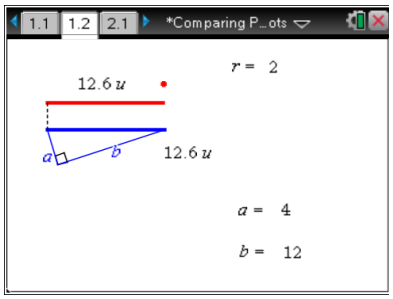
b. To better see how close the two lengths are, use the Measurement Tool to measure the circumference and the length of the hypotenuse and adjust the values of a and b until the measured lengths appear to be the same. Choose integer values for a and b. What values did you choose for a and b?

Answer: $a = 4, b = 12$ or $a = 12, b = 4$.

Tech Tip: In order to measure the length, use Length Measurement Tool by selecting **MENU > Measurement > Length**.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

2. When we measure the length of a segment, do we measure it approximately or exactly?



Answer: We use measurements to find length approximately, since measurements depend on how precise the measurement device is (or measurement tool in the TI-Nspire). We cannot determine if the value found by measurements are exact or approximate.

Teacher Tip: It is very important to discuss with the students why measurements are usually approximations of actual values and why we cannot determine if measured values are an exact value. It might be useful to let students use a ruler to measure the length of a segment. Measure the length to the nearest inch, the nearest half-inch, the nearest quarter-inch, and the nearest eighth-inch. Students will realize that their measurements are approximations that depend on the precision of the ruler.

3. When the measurements of the lengths of the circumference and hypotenuse are equal, does that mean that the two lengths are exactly the same? Explain.

Sample Answers: Because measurements are displayed with the precision of only 0.1, we cannot definitely know whether the lengths are equal exactly or approximately.

Teacher Tip: Discuss with students the difference between the length as a property of a geometric object and the length as a measurement. In this case, the measurement is an approximation of the actual length. This discussion will help students to recognize the fact that they need to use formulas to find exact values of lengths.

4. What types of numbers can be written for the measured lengths of the circumference and hypotenuse?

   Answer: The measured values are terminating decimals, so these are rational numbers.

   Teacher Tip: Help students realize that the measured values are decimal approximations and thus are rational numbers.

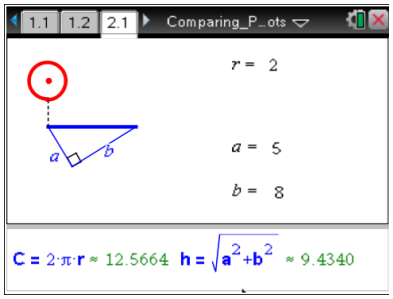
5. What is the formula for the circumference of a circle with a given radius? Use letter *C* for the circumference and *r* for the radius when writing the formula.

   Answer: The formula for the circumference is $C = 2\pi r$.

6. What is the formula that relates the length of the hypotenuse and the lengths of the legs of a right triangle? Use letter *h* for the hypotenuse and a and b for the legs when writing the formula.

   Answer: Using Pythagorean theorem, we get $h = \sqrt{a^2 + b^2}$.

Move to page 2.1.



7. Enter the values you found for r, a, and b onto Page 2.1. Record the computed values of the circumference and the hypotenuse displayed at the bottom of the page.

   Answer: For $r = 2$, $a = 4$, and $b = 12$, we get $C \approx 12.5664$ and $h \approx 12.6491$.

   Tech Tip: When students enter the values of the radius and the sides of the triangle onto Page 2.1, the lengths of the circumference and the hypotenuse are calculated and displayed at the bottom of Page 2.1. The calculations have greater precision than the measurements, and the values are displayed with up to 4 decimal places. The lengths that appeared to be

equal when expressed to the nearest 10th are no longer equal.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

8. Compare computed values of the length of the circumference and the length of the hypotenuse. Do they appear to be the same? Explain your findings.

Sample Answers: The computed values are not equal. Students' explanations will vary.

Teacher Tip: Let students explain their reasons for not getting the computed values of lengths to be equal in this situation. Do not provide explanations, but encourage students to make suggestions on further explorations. Ask students what else they can do to try to find a triangle that has the hypotenuse length that is *exactly* the same as the circumference length. Give students time to investigate the situation with various values of r, a, and b. Students can input any non-negative real values for the radius of the circle and the sides of the triangle. Most likely students will use decimals, but don't be surprised if some of the students decide to input irrational numbers. They can use displayed computed values at the bottom of Page 2.1 to adjust the values of variables to make the computed values as close to each other as possible. To save time, let students work in small groups and combine results of their investigations.

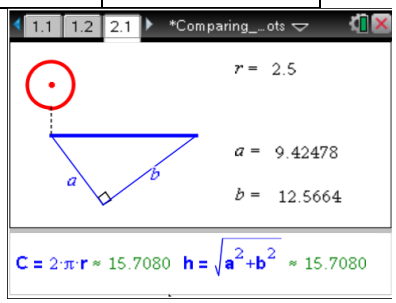
9. Explore further by changing the values of r, a, and b.
   - In order to keep the segments within the screen, keep the radius of the circle $r \leq 5$.
   - Unwrap the circle to compare the lengths visually and then try to adjust values of r, a, and b to make the circumference and hypotenuse appear to be the same length.
   - Record your findings in the table below.

Answer:

Radius, r	Side, a	Side, b	Computed circumference	Computed hypotenuse
2	4.1	11.9	12.5664	12.5865
2.3	9.9	10.5	14.4513	14.4312
3	2.37	18.7	18.8496	18.8496
2.5	3π	4π	15.708	15.708

$\frac{5}{\pi}$	6	8	10	10
$2\sqrt{3}$	6π	$2\sqrt{3}\,\pi$	21.7656	21.7656

Tech Tip: In order to keep the segments within the screen, keep the radius of the circle $r \le 5$. Students can input exact values for r, a, and b containing π and square root by pressing [π] or [ctrl] [x²] keys. The rational approximations of these values will be displayed on the screen.



TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 3 at the end of this lesson.

10. When the computed values for the lengths of the circumference and hypotenuse are equal, does it mean that the two lengths are exactly the same? Explain.

Answer: The computed values are decimal approximations of lengths, so we cannot state whether the lengths are exactly equal or not.

11. What types of numbers are the computed values of the lengths of the circumference and hypotenuse?

Answer: The computed values are terminating decimals, so these are rational numbers.

Teacher Tip: Ask them whether the calculator provides the exact or the approximated values. Help students realize that computed values are decimal approximations and thus are rational numbers.

12. What types of numbers are the exact values of the lengths of the circumference and hypotenuse?

Answer: The exact values could be rational or irrational because the formula for the circumference contains the irrational number π and the formula for the hypotenuse contains a square root that could lead to an irrational number.

13. Is it possible to find a circle and a right triangle so that the circumference and hypotenuse have *exactly* the same lengths? Why?

Answer: Yes, it is possible. Three examples are given in the last three rows of the table above. Students can check by plugging in the values into the formulas. For example, when $a = 3\pi$, $b = 4\pi$, $h = \sqrt{(3\pi)^2 + (4\pi)^2} = \sqrt{25\pi^2} = 5\pi$. For the circumference to equal 5π, $C = 2\pi r = 5\pi$, when $r = \frac{5}{2}$.

Teacher Tip: Emphasize that even though the calculations have higher precision, these are still decimal approximations of actual values. In order to confirm exact equality, students should use equations. If students have a hard time coming up with an example, help them by asking whether we can use irrational numbers for the radius or the sides of the right triangle. Provide them with one example, and let them come up with different examples. They can check their examples by inputting the exact values for the variables onto Page 2.1. Help students understand that they need to make the left and right sides of the equation $C = h$ or $2\pi r = \sqrt{a^2 + b^2}$ exactly the same. Let students observe that decimal approximations of the exact values are also equal.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to approximate the circumference of circles given the length of the radius and the hypotenuse of right triangles given the lengths of the sides.
- The circumference of a circle can be a rational or irrational number.
- The length of the hypotenuse can be a rational or irrational number.
- It is possible to make the length of the circumference of a circle and the length of the hypotenuse of a right triangle exactly equal by giving examples using rational and irrational numbers.

Assessment

Provide students with specific values of r, a, and b, and ask them to compare the circumference of the circle to the hypotenuse of the right triangle using calculations based on formulas. Students will find the decimal approximations for the two lengths and compare the values. Then let students check their calculations using the interactive model on Page 1.2 (or 2.1) of the TI-Nspire document in order to compare the lengths visually. Examples of problems that can be given to the students and lead to close or equal lengths:

1) $r = \pi, a = 8, b = 18$
2) $r = \sqrt{5}, a = 4\pi, b = 2\pi$
3) $r = \frac{10}{3}, a = 10\sqrt{2}, b = 10\sqrt{2}$

TI-Nspire Navigator

Note 1

Name of Feature: Screen Capture

Use *Screen Capture* to display student triangles. Select different triangles, and ask students to compare them. In Question 1a, make sure to discuss the reasons for different answers, such as approximations based on eyeballing. In Question 8, ask students to check different answers using evaluation of expressions to confirm that solutions offered by the students do lead to a true equation.

Note 2

Name of Feature: Quick Poll

Use the Open Response option in *Quick Poll* to collect student answers to Questions 1b, 5, and 6.

Note 3

Name of Feature: Live Presenter

Use *Live Presenter* for students to explain the methods they used to choose the values for r, a, and b. Ask students to explain their approaches. Make sure students demonstrate how they used the interactive model for initial comparison of lengths.