Area Measures and Right Triangles

Math Objectives

- Students will open or create a TI-Nspire document with a right triangle that has equilateral triangles on its sides. The areas of the equilateral triangles are measured and displayed on the screen.
- Students will investigate dependent and independent objects on a drawing, make a conjecture about the relationships between the areas of the equilateral triangles, and make a connection to the Pythagorean Theorem.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

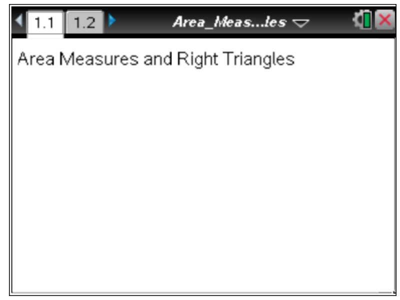
- equilateral triangle
- area of a triangle
- Pythagorean Theorem

About the Lesson

- The time varies for this activity depending on whether the TI-Nspire document is provided for or created by the students.
- Students can either create the TI-Nspire document by following the instructions given in *Area_Measures_and_Right_Triangles_Create.pdf,* or they can use the pre-constructed document entitled *Area_Measures_and_Right_Triangles.tns.*
- It is recommended that students create this TI-Nspire document so they can discuss dependent and independent objects in a dynamic geometry construction.
- Students should have knowledge of the Pythagorean Theorem before this activity.

TI-Nspire™ Navigator™ System

- Use Class Capture to observe students' work as they proceed through the activity.
- Use Live Presenter to have a student illustrate how he or she used a certain tool.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Files:

Create Instructions

- Area_Measures_and_Right_Triangles_Create.pdf

Student Activity

- Area_Measures_and_Right_Triangles_Student.pdf
- Area_Measures_and_Right_Triangles_Student.doc

TI-Nspire document

- Area_Measures_and_Right_Triangles.tns

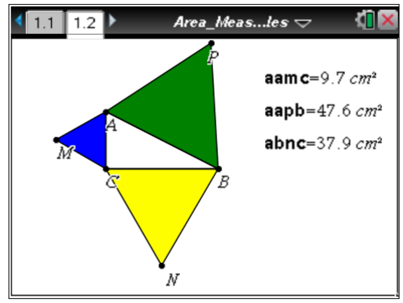
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Part 1: Making a conjecture

1. Which of the points *A, B, C, M, N,* and *P* cannot be moved? Explain why.

   <u>Answer:</u> Points *M*, *N*, and *P* cannot be moved. The location of point *M* is determined by points *A* and *C* (point *M* depends on points *A* and *C*). The location of point *N* is determined by points *B* and *C* (point *N* depends on points *B* and *C*). The location of point *P* is determined by points *A* and *B* (point *P* depends on points *A* and *B*).

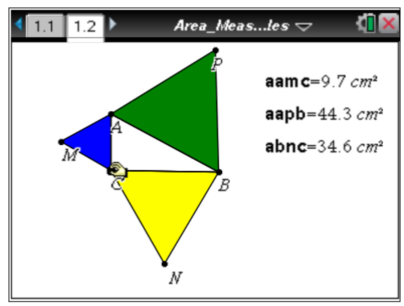


> **Teacher Tip:** If students have created this .tns file, they will be better able to explain why points *M, N, and P* do not move and why point *A* can move only along the perpendicular line.

2. Grab and drag point *C*. Observe the four triangles and the three area measures. What changes and what stays the same?

   <u>Answer:</u> Triangle *ACB* is changing shape and size, but it is always a right triangle because *AC* was constructed perpendicular to *CB*.

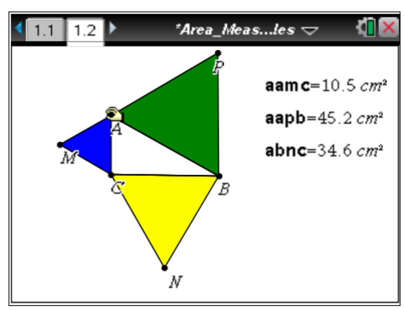
   All the triangles on the sides of *ACB* are still equilateral because the points *M, N,* and *P* were constructed to always be equidistant from the endpoints of segments *AC, CB,* and *AB,* respectively. The areas of triangles *APB* and *BNC* are changing. The area of triangle *AMC* stays same.



3. Grab and drag points *A* and *B*. Observe the four triangles and the three area measures. Compare your observations to those you made when dragging point *C*.

   <u>Answer:</u> Triangle *ACB* is always a right triangle, but its area and shape change.

   When one of the points *C*, *A*, or *B* is moved, all the triangles on the sides of *ACB* remain equilateral. Two of the areas change and one stays the same.



4. Make a conjecture about the relationship between the three area measures.

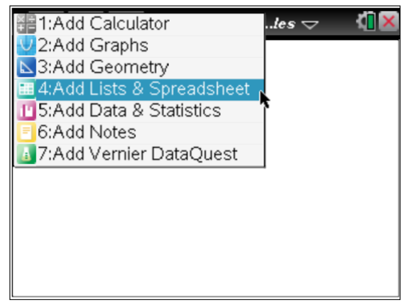
Answer: The sum of the areas of the equilateral triangles on the legs is equal to the area of the equilateral triangle on the hypotenuse.

> **Teacher Tip:** Stop at this point and have students share their conjectures. The remainder of this activity assumes you are verifying the conjecture above.

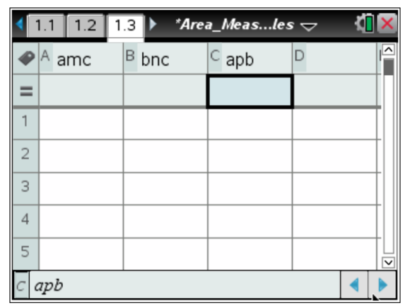
Part 2: Testing the conjecture

To test your conjecture, you will store some area measurements in a spreadsheet.

To insert a list and spreadsheet page: Press ctrl doc▾ **> Lists & Spreadsheet**.

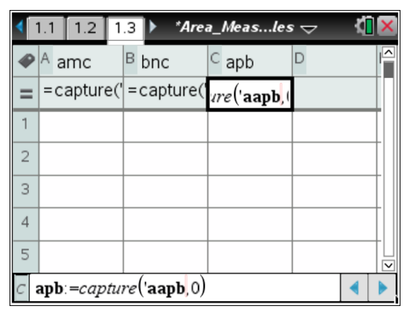


Name the columns: Highlight the cell above the formula (=) row in Column A and type *amc.* Press enter. Move to Column B above the formula row and type *bnc.* Press enter. Move to Column C above the formula row and type *apb.* Press enter.



Set up the manual data capture: Move to Column A in the formula row. Press **Menu > Data > Data Capture > Manual**. Press var and select *aamc* by pressing . Press enter.

Move to Column B in the formula row. Set the data capture as described above and select *abnc*. Move to Column C in the formula row. Set the data capture as described above and select a*apb*.



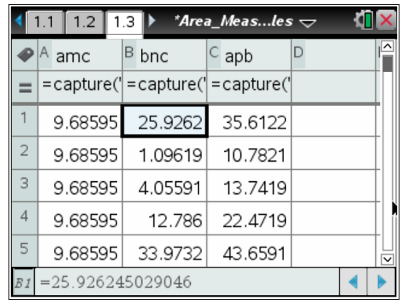
Press ctrl ◄ to return to page 1.2. Press ctrl . to collect data.

Drag points *A, B,* or *C*. Press ctrl . to collect data again. Drag a point and continue to collect at least four different data points.

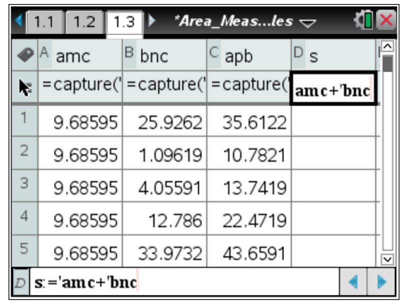
Press ctrl ▶ to return to page 1.3.

5. What relationship do you observe among the data lists?

Answer: The sum of the areas in Columns A and B equals the area stored in Column C.

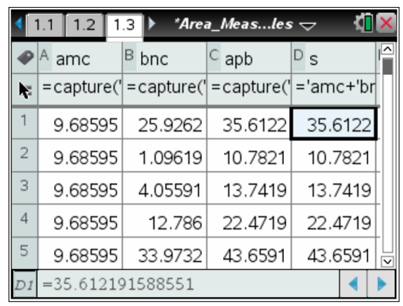


Move to Column D in the row above the formula row. Type S and press enter. Remain in the formula row of Column D. Press = var and select *amc*. Press + var and select *bnc*. Press enter to perform the calculation that adds columns A and B. Compare the values in Column D with the areas in Column C.



6. How does this data verify or disprove your conjecture?

Answer: The sums stored in Column D are the same as the area measures stored in Column C. The data verifies the conjecture.



7. Use what you know about the relationship of the measures of the legs of a 30°-60°-90° triangle to express the height of equilateral triangles with sides lengths *a, b,* and *c*.

Answer: $\dfrac{\sqrt{3}a}{2}, \dfrac{\sqrt{3}b}{2}, \dfrac{\sqrt{3}c}{2}$

8. If the equilateral triangles have side lengths *a, b*, and *c,* what are the areas of the three triangles?

Answer: $\dfrac{\sqrt{3}a^2}{4}, \dfrac{\sqrt{3}b^2}{4}, \dfrac{\sqrt{3}c^2}{4}$

9. Use the answers to question 8 to write an equation for the conjecture.

Answer: $\dfrac{\sqrt{3}a^2}{4} + \dfrac{\sqrt{3}b^2}{4} = \dfrac{\sqrt{3}c^2}{4}$

10. Divide the left and right sides of this equation by the GCF of all the terms. Do you recognize this equation? Where have you seen this equation before?

    Answer: $a^2 + b^2 = c^2$. This is a statement of the Pythagorean Theorem since *a* and *b* are the measures of the legs of a right triangle and *c* is the measure of the hypotenuse of a right triangle.

11. What other figures could be drawn on the sides of right triangles for which the following statement would be true?

    The sum of the areas of the two figures on the legs equals the area of the figure on the hypotenuse.

    Answer: Students may say regular polygons, or semicircles. Lead the discussion, if possible, to the generalization that if the figures are similar, this relationship will exist.