Making Limits Exist

About the Lesson

In this activity, students will graph piecewise functions and evaluate numerically and graphically the left hand limit and the right hand limit of the function as x approaches a given number, c. As a result, students will:

- Determine if a limit exists.
- Modify piecewise functions so that their limits do exist.

Vocabulary

- one-sided limits
- continuity

Teacher Preparation and Notes

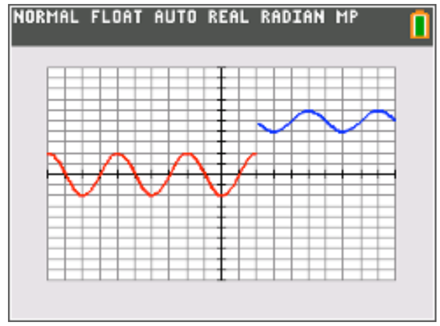
- Students should already have been introduced to one-sided limits.
- Students should know that a limit exists if and only if the left hand limit and the right hand limit are equal.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

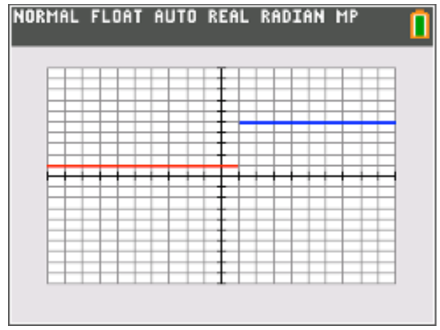
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Making_Limits_Exits_Student.pdf
- Making_Limits_Exist_Student.doc

Problem 1 – Linear Piecewise Function

Before changing the value of *a*, students will graphically estimate the limit of *f*(*x*) as *x* approaches 1 from the left and the right. Students will also use the table to numerically estimate the value of *a* that will ensure that the limit of *f*(*x*) as *x* approaches one exists.



Tech Tip: To set the domain for piecewise functions, each piece must be entered into its own equation line and be divided by its restricted domain.

1. Graphically, what do the following one-sided limits appear to be?

$$f(x)=\begin{cases}5, x\geq 1\\ 1, x<1\end{cases}$$

a. $\lim\limits_{x\to 1^-} f(x)\approx$ _____

Answer: 5

b. $\lim\limits_{x\to 1^+} f(x)\approx$ _____

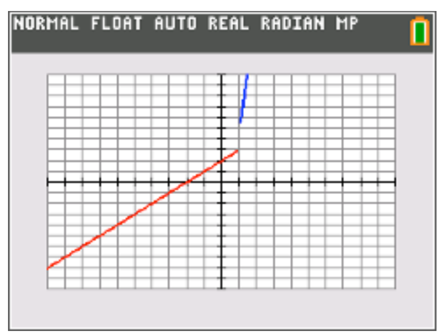
Answer: 1

2. After checking graphically, and numerically, what value of *a* resulted in *f*(*x*) being continuous?

Answer: *a* = 1

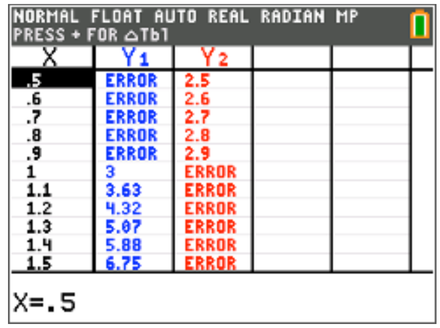
Problem 2 – Linear and Quadratic Piecewise Function

Problem 1 is repeated for a different function. Before changing the value of *a*, students will graphically estimate the limit of *g*(*x*) as *x* approaches 1 from the left and the right.



Students will use the table to numerically estimate the value of *a* that will ensure that the limit of *g(x)* as *x* approaches one exists.

Here the algebraic calculations for the left and right hand limits are to be shown.


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl

X	Y1	Y2
.5	ERROR	2.5
.6	ERROR	2.6
.7	ERROR	2.7
.8	ERROR	2.8
.9	ERROR	2.9
1	3	ERROR
1.1	3.63	ERROR
1.2	4.32	ERROR
1.3	5.07	ERROR
1.4	5.88	ERROR
1.5	6.75	ERROR

X=.5

3. Graphically and numerically, what do the following one-sided limits appear to be?

$$g(x)=\begin{cases}5\cdot x^2, x\geq 1\\ x+2, x<1\end{cases}$$

a. $\lim_{x\to 1^-} g(x) \approx$ _____

Answer: 3

b. $\lim_{x\to 1^+} g(x) \approx$ _____

Answer: 5

4. a. After checking graphically and numerically, what value of *a* resulted in *g*(x) being continuous?

Answer: $a = 3$

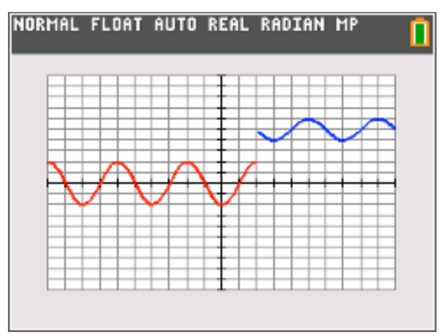
b. Show calculations of the left hand limit and the right hand limit to verify that your value for *a* makes the limit exist.

Answer:

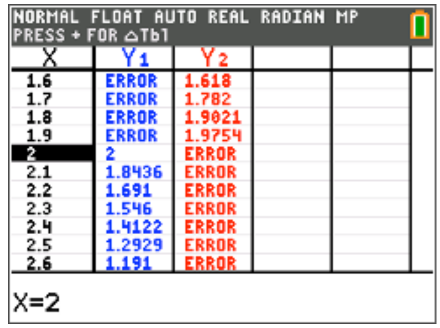
$$\lim_{x\to 1^-} g(x) = \lim_{x\to 1+} g(x)$$
$$1+2 = a\cdot 1^2$$
$$3 = a$$

Problem 3 – Trigonometric Piecewise Function

Problems 1 and 2 are repeated for a different function. Before changing the value of *a*, students will graphically estimate the limit of *h*(*x*) as *x* approaches 2 from the left and the right.



Students will use the table to numerically estimate the value of a that will ensure that the limit of $h(x)$ as x approaches two exists. Students should view the table near $x = 2$ instead of 1.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PRESS + FOR △Tbl

X	Y1	Y2		
1.6	ERROR	1.618		
1.7	ERROR	1.782		
1.8	ERROR	1.9021		
1.9	ERROR	1.9754		
2	2	ERROR		
2.1	1.8436	ERROR		
2.2	1.691	ERROR		
2.3	1.546	ERROR		
2.4	1.4122	ERROR		
2.5	1.2929	ERROR		
2.6	1.191	ERROR		

X=2

5. Graphically and numerically, what do the following one-sided limits appear to be?

$$h(x) = \begin{cases} 5 + 3\sin\left((x-4)\frac{\pi}{2}\right), x \ge 2 \\ 2\sin\left((x-1)\frac{\pi}{2}\right), \quad x < 2 \end{cases}$$

a. $\lim_{x \to 2^-} h(x) \approx$ _____

Answer: 2

b. $\lim_{x \to 2^+} h(x) \approx$ _____

Answer: 5

6. a. After checking graphically and numerically, what value of a resulted in h(x) being continuous?

Answer: $a = 2$

b. Show calculations of the left-hand limit and the right-hand limit to verify that your value for a makes the limit exist.

Answer:

$$\lim_{x \to 2^-} h(x) = \lim_{x \to 2^+} h(x)$$
$$2\sin\left(\frac{\pi}{2}(2-1)\right) = a + 3\sin\left(\frac{\pi}{2}(2-4)\right)$$
$$2\sin\left(\frac{\pi}{2}\right) = a + 3\sin(-\pi)$$
$$2 \cdot 1 = a + 3 \cdot 0$$
$$2 = a$$