# Radio Station KTNS



## Math Objectives

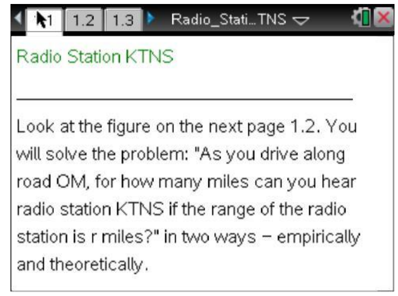
- Students will solve a problem experimentally by fitting a function to a set of data.
- Students will solve the same problem theoretically by making and verifying conjectures using algebraic and trigonometric methods.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).

## Vocabulary

- Law of Sines
- Law of Cosines

## About the Lesson

- This lesson involves determining the distance one can hear a radio station as a function of the range of the station.
- Note: Some portions of the activity require CAS functionality – TI-Nspire CAS Required.
- As a result, students will:
    - Solve the problem empirically by fitting a regression equation to a set of gathered data.
    - Solve the problem theoretically by finding an equation involving the Law of Cosines and the Law of Sines.




**TI-Nspire™ Technology Skills:**

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

**Tech Tips:**

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- Once a function has been graphed, the entry line can be shown by pressing [ctrl] [G]. The entry line can also be expanded or collapsed by clicking the chevron.

**Lesson Files:**

*Student Activity*

Radio_Station_KTNS_Student.pdf

Radio_Station_KTNS_Student.doc

*TI-Nspire document*

Radio_Station_KTNS.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

## Discussion Points and Possible Answers

Radio Station KTNS is located at point $P$ in the figure. The range of its signal is $r$ miles, meaning that people within $r$ miles of $P$ would be able to hear the station. You are driving along road $OM$ at an angle of 30° with $OP$. For how many miles, $d$, could you hear station KTNS?

In $\Delta PAB$, the Law of Cosines tells us that $d^2 = 2r^2 - 2r^2 \cdot \cos(\angle APB)$, so it is reasonable to assume that $d^2$ could be a linear function of $r^2$. To solve this problem, you will determine $d^2$ in terms of $r^2$ in two ways:

- Find an experimental model by gathering data and fitting an appropriate regression function to the data.
- Find a theoretical model using the Law of Sines, the Law of Cosines, and algebra.

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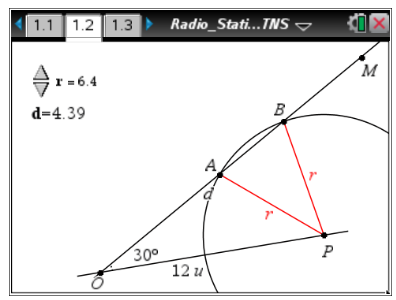


The figure is a scale drawing with 1 unit = 10 miles so that OP = 12 units or 120 miles.

1. In miles, the reasonable values of $r$ satisfy $k < r \le 120$. What is the value of $k$? Why?

   **<u>Answer:</u>** $k = 12 \cdot \sin(30°) = 6\ miles$ since the smallest value of $k$ occurs when $r$ is perpendicular to $OM$ and $d = 0$.

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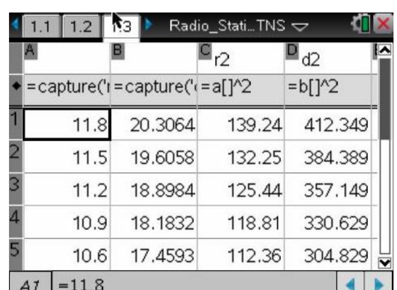

| A | B | C r2 | D d2 |
|---|---|---|---|
| =capture(' | =capture(' | =a[]^2 | =b[]^2 |
| 11.8 | 20.3064 | 139.24 | 412.349 |
| 11.5 | 19.6058 | 132.25 | 384.389 |
| 11.2 | 18.8984 | 125.44 | 357.149 |
| 10.9 | 18.1832 | 118.81 | 330.629 |
| 10.6 | 17.4593 | 112.36 | 304.829 |

Using the slider, the following data has been gathered in the spreadsheet in the four columns: $rad(r)$ $\quad dis(d)$ $\quad r2 = r^2$ $\quad d2 = d^2$

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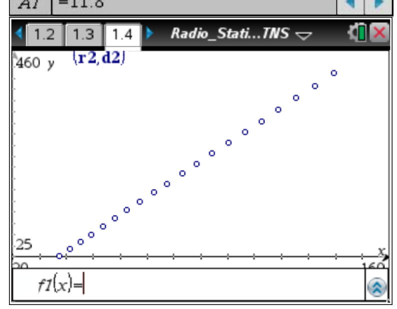


A scatterplot of the data has been drawn on this page.

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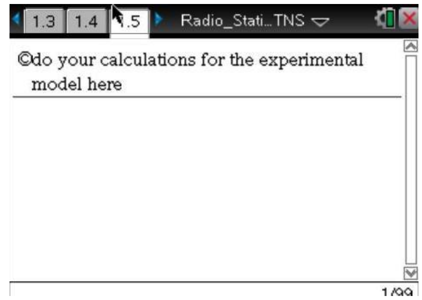


2. Fit a linear regression function to the data with $x = r2$ and $y = d2$ in units. Select **MENU > Statistics > Stat Calculations > Linear Regression (mx+b)**. with $r2$ for X List, $d2$ for Y List, and **Save RegEqn** to: $f1$.

   **Answer:** $d^2 = 4r^2 - 144.611$.

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3. Plot the regression equation on the scatterplot, and note how well it fits. Open the entry line, move back up to $f1(x)$, and press enter. According to this linear model, for how many miles, $d$, could you hear the station if $r = 90$ miles?
   Hint: Remember $r$ = 9 units corresponds to $r$ = 90 miles.

   **Answer:** $\sqrt{4 \cdot 9^2 - 144.61} \cdot 10 = 133.94$ miles.

> **Teacher Tip:** Students could use Scratchpad or the Calculator page to compute their answers for #3.

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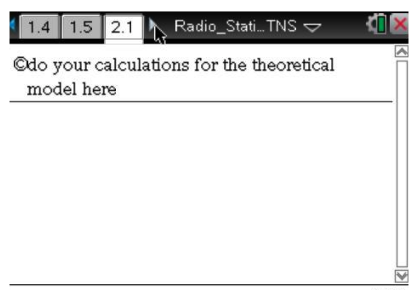
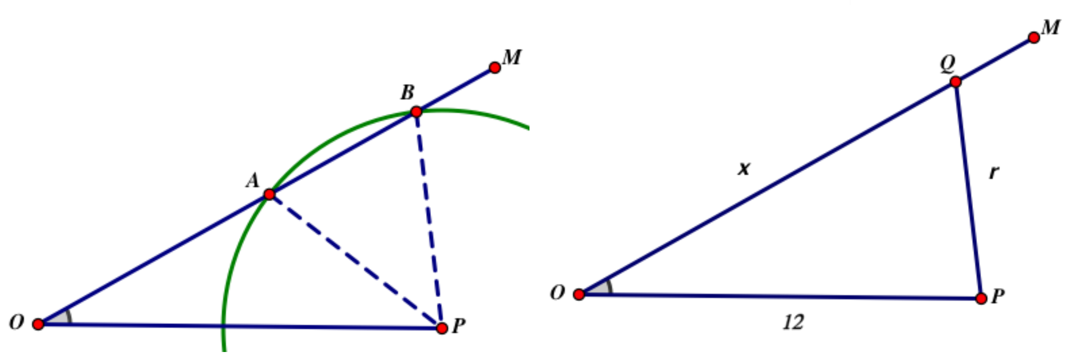



**Theoretical Model**

Find the theoretical function expressing $d^2$ in terms of $r^2$ by completing the argument below.




4. The figure for this problem shows an example of an ambiguous case of the Law of Sines since there are two triangles with two sides $OP$ = 12, $r$, and the non-included angle of 30°.

Consequently, if we apply the Law of Cosines to a triangle with sides $OP$ = 12, $r$ , $x$ and angle 30°, we obtain the equation:

_________________________________ = 0.

On the scale drawing, then, the two solutions for $x$ are $OA$ and $OB$, and the distance, $d$, is $d = OB - OA$.

**Sample Answers:** By the Law of Cosines, $r^2 = x^2 + 12^2 - 2 \cdot 12 \cdot x \cdot \cos(30°)$, so that the desired equation is $x^2 - 12\sqrt{3} \cdot x + (144 - r^2) = 0$.

5. a. Find the two solutions for $x$ of this equation.______________________________ .
Hint: You can use "solve" command. Both solutions will be functions of $r^2$

**Sample Answers:** Using paper-and-pencil or $solve(x^2 - 12\sqrt{3}x + 144 - r^2 = 0, x)$, the two solutions are $x = 6\sqrt{3} - \sqrt{r^2 - 36}$ and $x = 6\sqrt{3} + \sqrt{r^2 - 36}$.

b. Find the difference of the two solutions and express $d^2$ in terms of $r^2$ in units:

$d^2$ = ________________________

**Answer:** The difference is $d = 2\sqrt{r^2 - 36} = \sqrt{4r^2 - 144}$ so that $d^2 = 4r^2 - 144$.

6. How does your theoretical equation compare to the regression equation?

**Answer:** They are essentially the same with only a small difference in the constant terms.

7. According to this theoretical model, for how many miles, $d$, could you hear the station if $r = 90$ miles?
Hint: Remember $r$ = 9 units corresponds to $r$ = 90 miles.

**Answer:** $\sqrt{4 \cdot 9^2 - 144.} \cdot 10 = 134.16$ miles.

8. Suppose the angle between the two roads $OP$ and $OM$ is changed to $\theta°$. Express $d^2$ in terms of $r^2$ and $\theta$:

   $d^2$ = ________________________

   **Answer:** We want to find the square of the difference of the two solutions of $x^2 - 24x \cdot \cos\theta + (144 - r^2) = 0$. If we use 'paper-and-pencil', we will probably obtain $d^2 = 4r^2 + 576(\cos^2\theta - 1)$. Using $solve(x^2 - 24 * \cos\theta + 144 - r^2 = 0, x)$ and some rewriting yields $d^2 = 4r^2 - 576\sin^2\theta$.

   **Teacher Tip:** Ask students why these two solutions are equivalent.

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## Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to interpret a scale drawing.
- How to fit a linear regression equation to a set of data.
- Setting up and solving an equation involving the Law of Cosines and interpreting the solutions.

 