

# Antonio's Pizza Palace

(source: [http://mathforum.org/workshops/usi/pascal/pizza\\_pascal.html](http://mathforum.org/workshops/usi/pascal/pizza_pascal.html))

It's Friday night and the Pizza Palace is more crowded than usual. At the counter the Pascalini's are trying to order a large pizza, but can't agree on what topping(s) to select.

Antonio, behind the counter, says, "I only have 8 different toppings. It can't be that hard to make up your mind. How many different pizzas could that be?"

"Well, we could get a plain pizza with no toppings," says Mr. Pascalini.

"Or we could get a pizza with all 8 toppings," says Mrs. Pascalini.

"What about a pizza with extra cheese and green peppers?" asks Pepe.

"You're not helping!" Antonio yells at Pepe. "Get back to work."

As Pepe starts to clear off the nearest table, he mumbles to himself, "or a pizza with anchovies, extra cheese, mushrooms, and olives."

Antonio hands an order pad to Mr. Pascalini and says, "When you decide, write it down and I'll make it." Then he helps the next people in line, who know what they want: a large pizza with mushrooms, green peppers and tomatoes.

How many different pizzas can be ordered at the Pizza Palace if a pizza can be selected with any combination of the following toppings: anchovies, extra cheese, green peppers, mushrooms, olives, pepperoni, sausage, and tomatoes?

**As you solve this problem, look for patterns and answer the following questions:**

1. How many different pizzas can you order with only one topping?
2. How many different pizzas can you order each with seven toppings?
3. Are the number of one-topping pizzas and the number of seven-topping pizzas related?  
(Why or why not?)
4. How many different pizzas can you order with two toppings?
5. How many different pizzas can you order with six toppings?
6. Are the number of two-topping pizzas and the number of six-topping pizzas related?  
(Why or why not?)
7. Can you find these numbers in Pascal's triangle?
8. Can you use Pascal's triangle to help you find the number of pizzas that can be ordered with three, four, or five toppings?
9. In all, how many different pizzas can be ordered?

**Let's examine a faster way to find the total number of possibilities.**

Antonio could have helped the Pascalini's decide if he had asked the following questions:

- Do you want anchovies?
- Do you want extra cheese?
- Do you want green peppers?
- Do you want mushrooms?
- Do you want olives?
- Do you want pepperoni?
- Do you want sausage?
- Do you want tomatoes?

1. How many possible answers are there to each of the above questions?
2. How many questions are there?
3. Use your answers to the two questions above to determine total number of possibilities. Express your solution both with and without a power.
4. Antonio has decided to add four more toppings to his menu: chicken, ground beef, ham, and pineapple. How many different pizzas can be ordered now? Explain two different methods for finding the solution.