

Similarity and Dilations - Discover Properties of Similar Figures

Geometry Major Topics: Similar Figures and the Role of Dilations

NCTM Principles and Standards:
Content Standards

Geometry:

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make conjectures about them, and solve problems involving them.

Apply transformations and use symmetry to analyze mathematical situations.

Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices.

Use various representations to help understand the effects of simple transformations and their compositions.

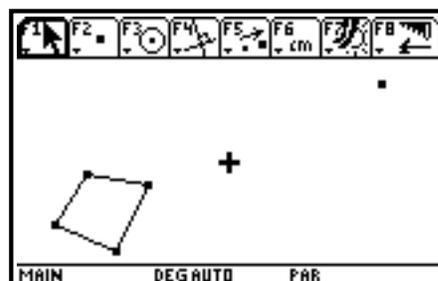
Objectives: This lesson has students discover properties of similar figures. Using the CABRI Geometry II Application for the TI-89 Calculator, students use dilations to dynamically manipulate figures in order to discover properties of similar figures. This lesson can be used as a teacher directed activity or can be given to the students as a discovery activity. If the students are familiar with the CABRI Geometry II Application, this activity should take one 50-minute class period.

Materials: TI-89 Graphing Calculator, CABRI Geometry II Application

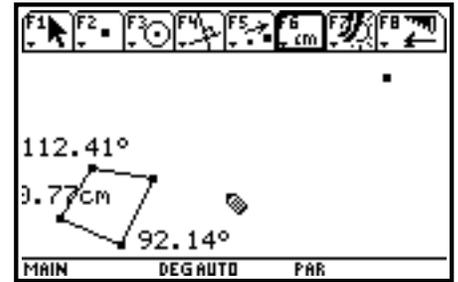
Prerequisites: Students should have a working knowledge of the CABRI Geometry II software, especially the transformational geometry commands, and have an intuitive idea that similar figures are figures that have the same shape but not necessarily the same size.

Procedure:

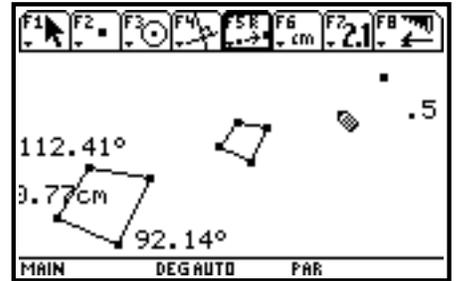
Activity 1: Open the CABRI Geometry II application on the TI-89 by pressing the APPS button and choosing Cabri Geometry. Open a NEW file and choose an appropriate variable name, such as similar. From the home screen, press F3 and choose **4:Polygon**. Construct a convex quadrilateral in the lower left corner of the screen. Press F2 and choose **1:Point**. Construct a point in the upper right corner of the screen.



Press F6 and choose **3:Angle**. Measure two different angles of the quadrilateral. Press F6 and choose **1:Distance & Length**. Measure the length of one side of the quadrilateral by measuring the distance between two consecutive vertices.



Press F7 and choose **6:Numerical Edit**. Move the cursor to a location near the point in the upper right corner of the screen and press ENTER. This creates a numerical edit box. Type in the decimal $.5$. Press F5 and choose **3:Dilation**. Move the cursor to the quadrilateral; when the message "dilate this polygon" appears, press ENTER. This establishes the quadrilateral as the object to be dilated. Move the cursor to the point in the upper right corner of the screen; when the message "with respect to this point" appears, press ENTER. This establishes the point as the center of dilation. Move the cursor to the decimal; when the message "using this factor" appears, press ENTER.



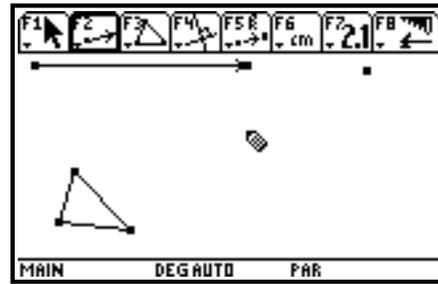
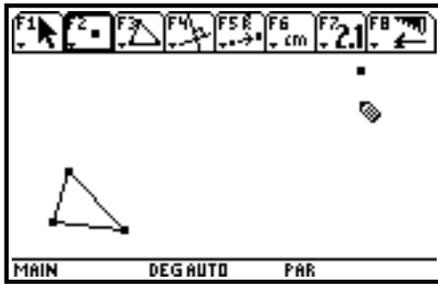
The original figure has been dilated toward the center of dilation by a factor of $.5$. The "new" quadrilateral is similar to the original quadrilateral, since dilations produce similar figures.

Press F6 and choose **3:Angle**. Measure the two angles on the "new" quadrilateral that correspond to the angles measured on the original quadrilateral. Press F6 and choose **1:Distance & Length**. Measure the side of the "new" quadrilateral that corresponds to the side measured on the original quadrilateral. Make a conjecture about angle measures and lengths of similar figures.

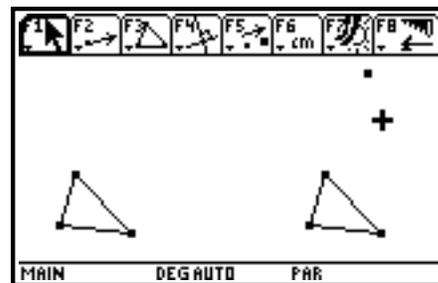
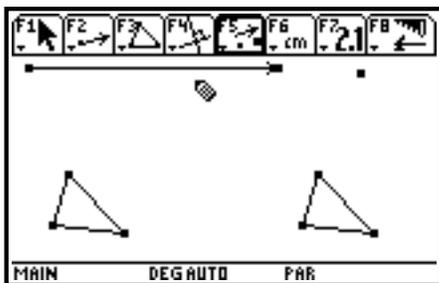
Test this conjecture by performing several more dilations. Press F7 and choose **6:Numerical Edit**. Move the cursor to the decimal $.5$; when the message "this number" appears, press ENTER. This will create an edit box around the number, allowing you to change the number. Delete the number $.5$ and type in the decimal $.3$. Observe the "changes" in the angle measures and the lengths. Examine various decimals smaller than 1, and if necessary, refine your conjecture.

Teacher Note: The conjecture should be "similar figures have congruent angles and sides that are proportional." The conjecture about congruent angles is readily seen. The conjecture about sides being proportional is not as easily seen. Activity 2 will further develop this idea. If the students try using a decimal greater than 1, the dilation will still occur but might not be easily seen; since this enlarges the figure, the image quadrilateral might not appear on the viewing screen. Students should experiment with various dilation factors, including 1 , 0 , and possibly negative numbers as factors.

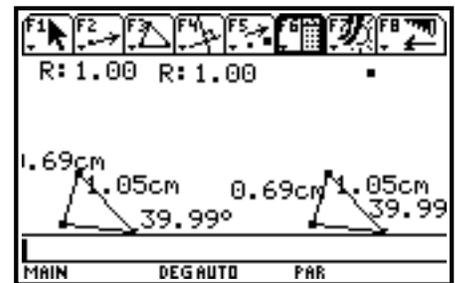
Activity 2: Press F8 and choose **8:Clear All**. Press ENTER to clear the screen for the next construction. Press F3 and choose **3:Triangle**. Construct a scalene acute triangle in the lower left corner of the screen. Press F2 and choose **1:Point**. Construct a point in the upper right corner of the screen. Press F2 and choose **7:Vector**. Construct a horizontal vector in the upper left corner of the screen. Your screens should be similar to the ones below.



Press F5 and choose **1:Translation**. Move the cursor to the triangle; when the message "translate this triangle" appears, press ENTER. This establishes the triangle as the object to be translated. Move the cursor to the vector; when the message "by this vector" appears, press ENTER. This translates the triangle in the direction of the vector by the magnitude of the vector. If necessary, drag the head of the vector so the image triangle is positioned in the lower right corner of the screen. Press F7 and choose **1:Hide / Show**. Hide the vector and each of its end points. Your screens should be similar to the ones below.



Press F6 and choose **3:Angle**. Measure an angle of the original triangle and the corresponding angle of the image triangle. The measures should be the same, since the figures are congruent. Press F6 and choose **1: Distance & Length**. Measure two of the sides of the original triangle and the corresponding sides of the image triangle, by measuring the distances between the vertices. The measures should be the same, since the figures are congruent. Press F6 and choose **6:Calculate**. Divide the lengths of the image sides by the corresponding lengths of the original sides. Since these measures are equal, the ratios should both be 1.

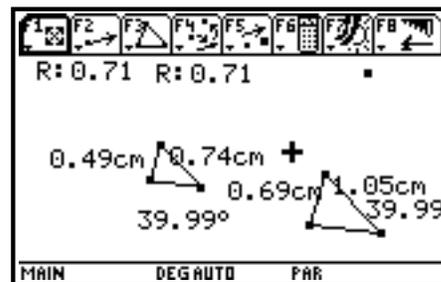


Press F4 and choose **B:Redefine Object**. Move the cursor to a vertex of the image triangle; when the message "this point" appears, press ENTER.



Choose **1:Point** and press ENTER. Although the screen appears not to have changed, this point has now been redefined as a "free point", with no connection to the original triangle. Repeat this procedure with the other two vertices of the image triangle. The entire triangle has now been redefined and is no longer connected to the original triangle by the vector translation.

Press F1 and choose **3:Dilate**. Move the cursor to the point in the upper right corner of the screen and press ENTER. This will establish this point as the center of dilation. Move the cursor to the triangle and drag it toward the center of dilation. Observe the changes in the angle measures, the changes in the measure of the lengths of the sides, and the changes in the ratios.



Make a conjecture about similar figures and test this conjecture by further dilations.

Teacher Note: The conjecture should still be "similar figure have congruent angles and sides that are proportional." The conjecture about angles is reconfirmed in this activity, and the conjecture about proportional sides is more readily seen. Although the side measures and the ratios change, the ratios of corresponding sides remain the same. If further investigation is needed, the third sides of the triangles could be measured and their ratio could be found. In addition, perimeters, lengths of corresponding medians, or any other relevant measures could be found and their ratios computed.

Ray Klein
 Glenbard West High School
 Glen Ellyn, IL
 RKlein9019@aol.com

Ilene Hamilton
 Adlai Stevenson High School
 Lincolnshire, IL
 ihamilto@district125.k12.il.us