Limits – ID: 8997

By Steve Ouellette

Time required
45 minutes

Activity Overview

Students will investigate finding the value of limits using graphical and numerical methods. They will also learn that a limit can exist at points where there is a hole or removable discontinuity. The concept of left- and right-sided limits will also be explored, as well as some situations in which limits do not exist.

Concepts

- *limits*

Teacher Preparation

This investigation offers an opportunity for students to use graphical and numerical methods to explore the concept of a limit.

- *Students may have an informal understanding of what a limit is. This activity will allow students to build on this understanding and develop a definition for limit. In particular, students will learn that, for y = f(x),*
  - *the value of* $\lim_{x \to a} f(x)$ *does not need to equal f(a), and* $\lim_{x \to a} f(x)$ *may exist even if f(a) does not exist.*
  - *a limit can be defined "from the right" or "from the left."*
  - *some limits do not exist.*
  - *some limits can be evaluated by direct substitution, whereas others must be evaluated using graphical, numerical, or algebraic methods.*
- *The screenshots on pages 3–5 demonstrate expected student results. Refer to the screenshots on page 6 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "8997" in the quick search box.***

Classroom Management

- *This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. The student worksheet is intended to guide students through the main ideas of the activity and provide a place to record their observations.*
- *For the most part, students will manipulate pre-made sketches, rather than create their own constructions. Therefore, a basic working knowledge of the TI-Nspire handheld is sufficient.*
- *It should be noted that infinite limits are not explored in this activity.*

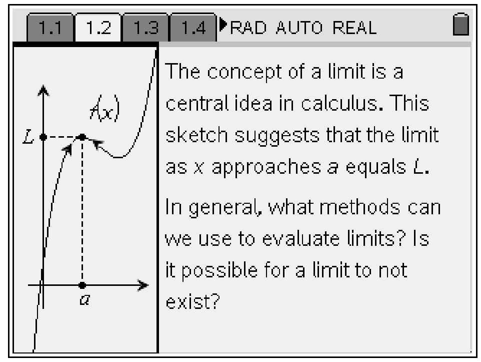
- *The last example that students investigate,* $\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}$*, suggests the need for evaluating limits algebraically. As an extension to this activity, provide students with examples in which expressions can be simplified algebraically and then evaluated using direct substitution. For example, have students evaluate* $\lim_{x \to 1} \frac{x^2-1}{x-1} = \lim_{x \to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x \to 1}(x+1) = 2$*. Make sure that students realize that* $x+1$ *and* $\frac{x^2-1}{x-1}$ *are not equivalent expressions at x = 1; however, their limits are identical since we are interested in the value of these expressions near 1. The expression* $\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}$ *can be simplified by multiplying by* $\frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3}$*, an expression equal to one that uses the conjugate of the numerator of the limit expression.*

- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the activity is completed successfully.*

TI-Nspire™ Applications

Calculator, Graphs & Geometry, Lists & Spreadsheet, Notes

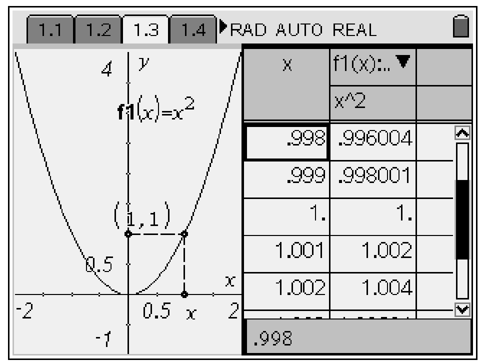
Two focus questions define this activity: *How can we use graphical and numerical methods to determine the value of a limit? Under what circumstances might a limit not exist?*



After some discussion, students should recognize that, for the sketch shown on page 1.2, $\lim_{x \to a} f(x)$ appears to be equal to $L = f(a)$. Explain to students that they will explore some situations that are not as straightforward as this example. Also mention that their work on this activity will help them understand the definition of a limit.

Problem 1 – Using graphical and numerical methods to find the value of a limit

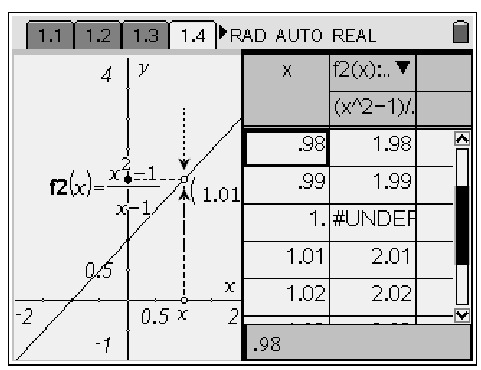
Step 1: This first example is similar to the one shown on page 1.2. While the value of the limit can be found by direct substitution ($\lim_{x \to 1} \mathbf{f1}(x) = 1^2 = 1$), it is important that students go through the process of investigating $\mathbf{f1}(x)$ for values of *x* very close to 1.



The screen shown to the right provides both graphical and numerical support for the value of the limit.

Look for opportunities to emphasize the need to focus on investigating $\mathbf{f1}(x)$ for values of *x* very close to 1.

Step 2: On page 1.4, students will perform a similar analysis as they did on page 1.3. This time, however, they <u>must</u> explore the value of $\mathbf{f2}(x)$ for values of *x* near 1 because the function is not defined for *x* = 1.



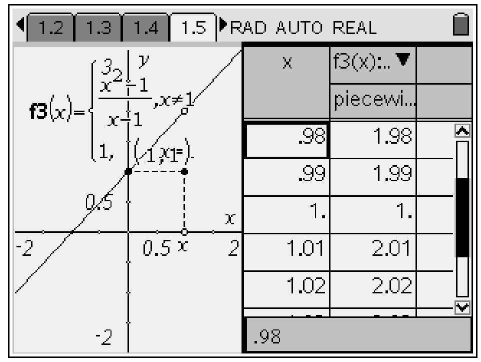
Make sure that students understand that a limit at *x* = *a* <u>can</u> exist even if $f(a)$ does not.

Explain to students that this function is said to have a *hole* or *removable discontinuity* at *x* = 1.

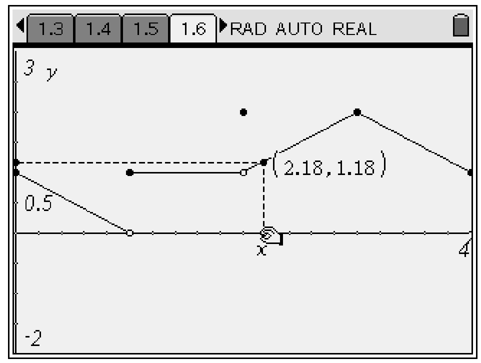
Step 3: The example on page 1.5 is very similar to that on the previous page, except that $\mathbf{f3}(x)$ does exist at $x = 1$.

Be sure students understand that $\lim\limits_{x\to 1} \mathbf{f3}(x) = 2$ even though $\mathbf{f3}(1) = 1$.

Explain that this function also has a *removable discontinuity* at $x = 1$.

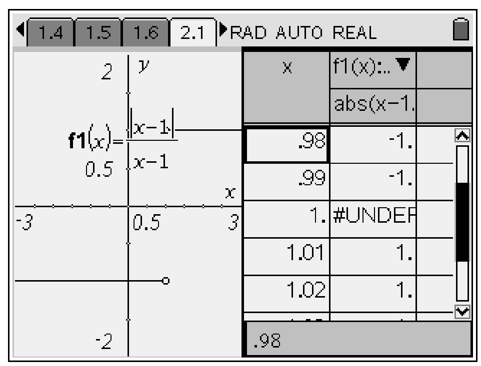


Step 4: This example allows students to synthesize their findings to this point. It also provides students with an opportunity to explore one-sided limits, as well as the idea that a limit only exists if the left and right-sided limits approach the same number.

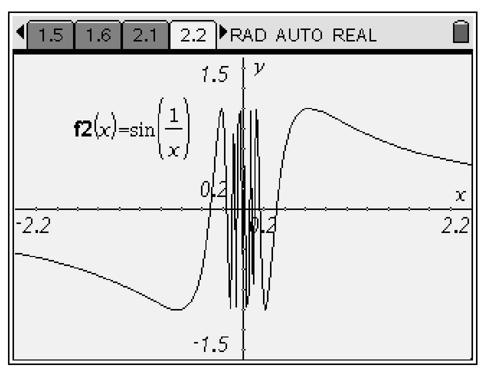


Problem 2 – Exploring limits that do not exist

Step 1: This exercise will help clarify that *jump discontinuities,* such as those encountered with step functions, have places where limits do not exist. Make sure students realize that, while $\lim\limits_{x\to 1} \mathbf{f1}(x)$ does <u>not</u> exist, $\lim\limits_{x\to 1^-} \mathbf{f1}(x)$ and $\lim\limits_{x\to 1^+} \mathbf{f1}(x)$ <u>do</u> exist and are equal to –1 and 1, respectively.

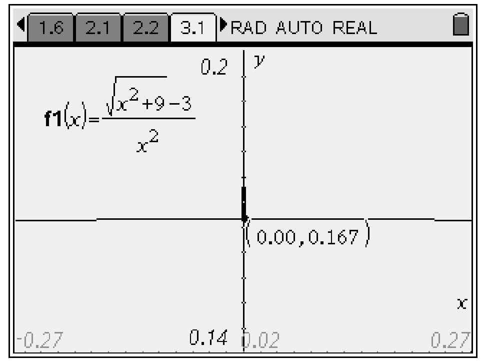


Step 2: On page 2.2, students will see another example of how a limit at a given point does not exist. In this case, the function $\mathbf{f2}(x) = \sin\left(\frac{1}{x}\right)$ continues to oscillate between –1 and 1 rather than approach a fixed number.



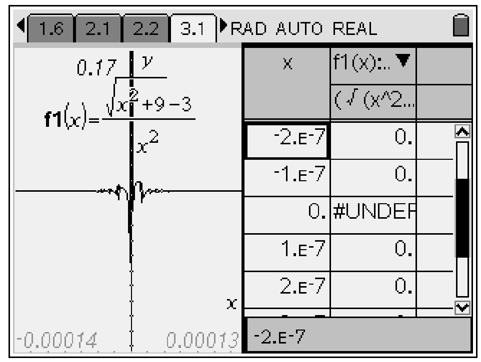
Problem 3 – Using other methods to find the value of a limit

Step 1: Page 3.1 is intended to alert students to the fact that graphical and numerical methods of analyzing a limit sometimes produce contradictory results.

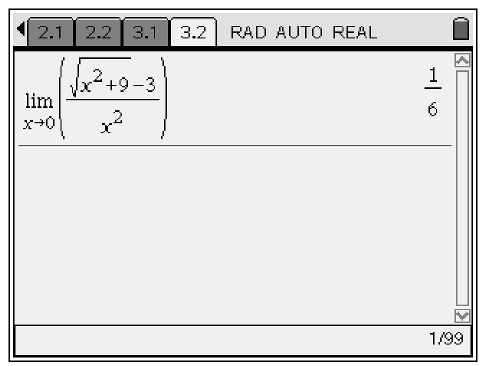


The screen at right shows, graphically and numerically, that the limit as x approaches 0 appears to be $\frac{1}{6}$.

The next screen shows that erroneous values occur after zooming in many more times. This observation will help students realize the need to use algebraic methods to evaluate limits.



Step 2: The *Calculator* application shown at right shows how the **Limit** command can be used to evaluate the limit of the function from page 3.1. Students are also asked to use this command to verify the value of the other limits that were studied in this activity.



This presents a good opportunity to introduce methods for evaluating limits analytically.

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(Student)TI-Nspire File: *CalcAct08_Limits_EN.tns*