Assessment Task Solutions
TI-30XB MultiView™: Factors in their Prime

ATS

1. What is the smallest number that has three different prime factors?

2, 3 and 5 are the smallest prime numbers, therefore 2 x 3 x 5 = 30 is the smallest number with three different prime factors.

2. What is the smallest number that has the factors:

a) 2, 3, 5 and 7
2, 3, 5 and 7 are all prime; therefore: $2 \times 3 \times 5 \times 7 = 210$ is the smallest number that contains all these factors.

b) 2, 3, 5, 6 and 7
$2 \times 3 \times 5 \times 7 = 210$

3. Comment on your answers to parts (a) and (b) in the previous question:

The number 6 is redundant in the computation since the prime factorisation of 6 is 2×3. Any number that has 2 and 3 in its prime factorisation will have 6 as one of its factors. The answer is therefore 210 for both parts (a) and (b).

4. What is the smallest number that has the factors:

a) 2, 4, 6 and 7
$2 \times 2 \times 3 \times 7 = 84$

b) 2, 4, 7 and 12
$2 \times 2 \times 3 \times 7 = 84$

c) Comment on your answers to parts (a) and (b). Include a discussion about how you obtained your answers.
The prime factorisation of 84 is $2^2 \times 3 \times 7$. The prime factorisation can be used to produce the factors 4 and 12. Students may refer to 'common' factors in their answers or may talk about 'working backwards'. This often includes multiplying all the numbers: $2 \times 4 \times 6 \times 7 = 336$ and gradually reducing this quantity. When a student uses this approach they have a lesser understanding of the significance of prime factorisation.

5. What is the smallest number that contains all the numbers from 1 to 20 as its factors?

232792560 has prime factorisation: $2^4 \times 3^2 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19$. Students need to identify all the prime numbers between 1 and 20: 1, 2, 3, 5, 7, 11, 13, 17 and 19. In addition to these factors, some need to be repeated in order to obtain the remaining numbers such as $16 = 2^4$. Another alternative is for students to write down the prime factorisation of the numbers from 1 to 20 and use this information to determine the answer.

6. The following numbers are relatively large. A systematic approach should be used to identify the prime factors. Mathematical shorthand is also appropriate for the prime factor expression.

Write down the *prime factorisation* of each of the following:

a) $512 = 2^9$
b) $497664 = 2^{11} \times 3^5$
c) $6561 = 3^8$
d) $119744 = 2^6 \times 1871$
e) $31104 = 2^7 \times 3^5$
f) $15625000 = 2^3 \times 5^9$

7. Explain any strategies you used in question 6 to determine the prime factorisation of each number.

Repeated division using basic number facts as a guide.

For example:
- *15625000 ÷ 5 = 3125000*
- *3125000 ÷ 5 = 625000*
- *625000 ÷ 5 = 125000*

This process can continue whilst the last digit of the number is either a 0 or a 5.

8. Write down the prime factorisation of your home phone number.
Does anyone in your class have a prime phone number?

Answers will vary. Answers can be checked using a CAS calculator or a website that completes prime factorisation, such as: http://www.cryptographic.co.uk/factorise.html

9. Write down the prime factorisation of this year (and subsequent years).

$2009 = 7^2 \times 41$
$2010 = 2 \times 3 \times 5 \times 6 \times 7$
2011 = Prime
$2012 = 2^2 \times 503$
$2013 = 3 \times 11 \times 61$

10. When is the next prime year?

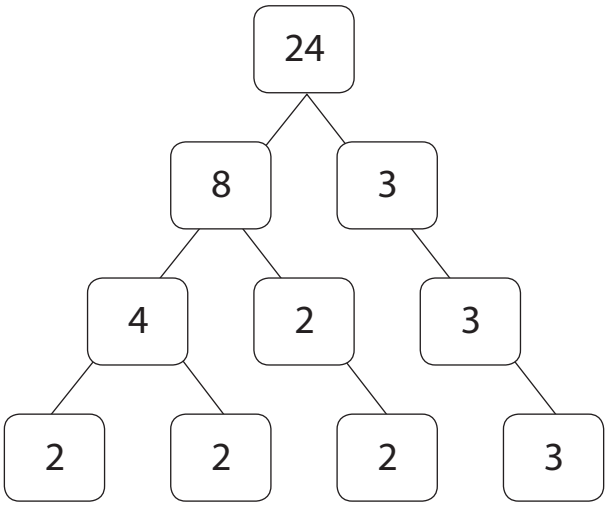
Next prime year is: 2011, followed closely by 2017 and then 2027. Note also that 2003 was a prime year.

Student Worksheet 2 Solutions
TI-30XB MultiView™: Factor Tree

WS2

A factor tree is a visual way to represent the factors of a number. The last row of factors have something in common, they are all prime numbers. Factor trees are helpful in finding the 'prime factors' of a number.

A factor tree for the number 24 is shown below:



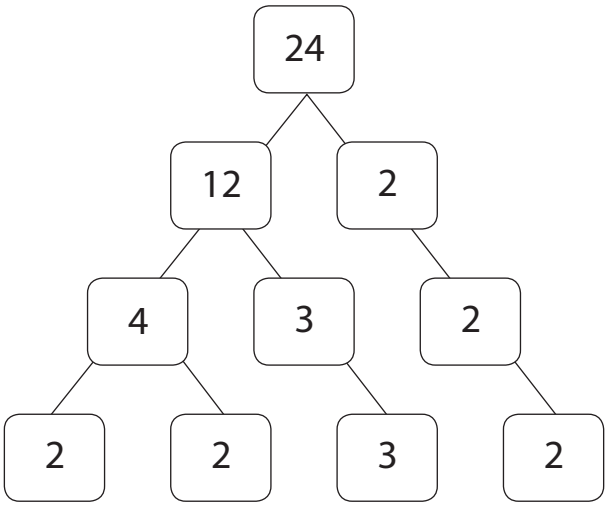
Observations:

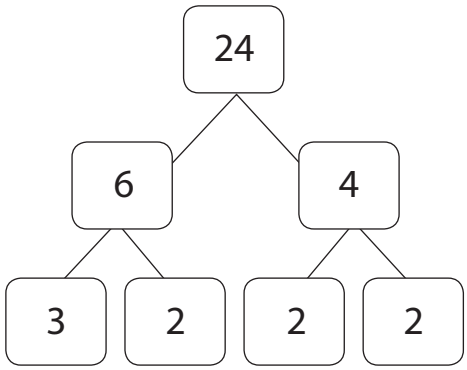
1. The first line: 8×3 is not the only possibility.
2. The product of each line is 24:

   $8 \times 3 = 24$
   $4 \times 2 \times 3 = 24$
   $2 \times 2 \times 2 \times 3 = 24$

3. The last line consists of prime numbers.[1]

1. Fill in the missing numbers for these factor trees: *Shown below*





2. Write down any observations you can make about the bottom row of factors for each of the factor trees for the number 24?

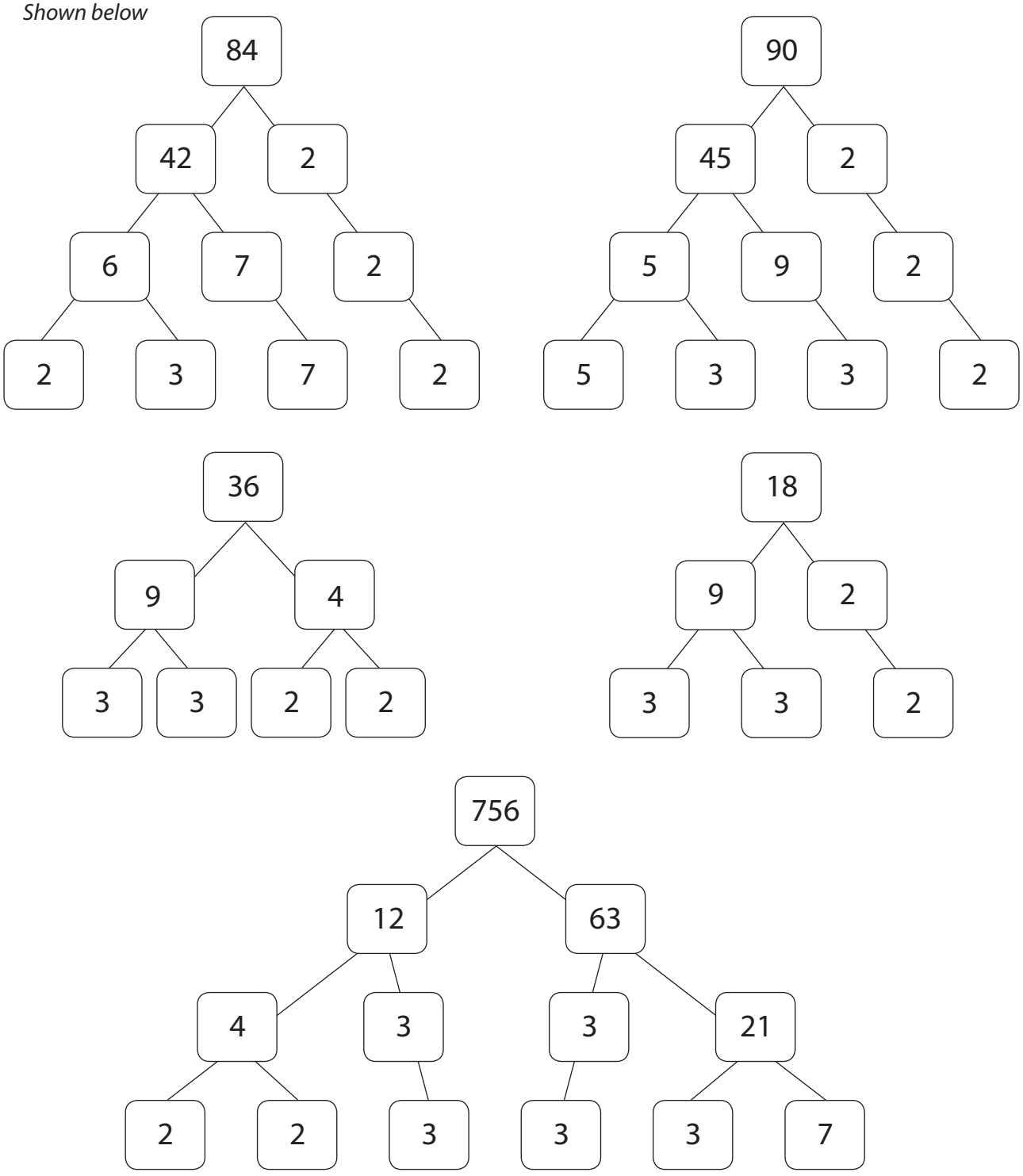
- *The bottom row has the same set of numbers.*
- *The numbers in the bottom row are all prime. (Prime factorisation)*

Student Worksheet 2 Solutions
TI-30XB MultiView™: Factor Tree

WS2

3. Complete the factorisation trees for each of the following:
Check that the product of the factors in each line equals the original number.

Shown below



4. Complete the factorisation trees for each of the following:

Answers will vary… but the prime factorisation will be the same:

a) $21 = 7 \times 3$
b) $45 = 3 \times 3 \times 5$ or $3^2 \times 5$
c) $60 = 2 \times 2 \times 3 \times 5$ or $2^2 \times 3 \times 5$
d) $49 = 7 \times 7$ or 7^2
e) $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ or 2^6
f) $81 = 3 \times 3 \times 3 \times 3$ or 3^4

5. Write down the original number for each of the following prime factor expressions:

a) $2 \times 2 \times 2 \times 3 = 24$
b) $3 \times 3 \times 5 \times 7 = 315$
c) $2 \times 2 \times 3 \times 5 = 60$
d) $2 \times 3 \times 3 \times 11 = 198$
e) $5 \times 7 \times 11 = 385$
f) $13 \times 17 = 91$

6. Write down all the prime numbers between 1 and 60.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59.

7. Write down the prime factorisation of each of the following:

a) $42 = 2 \times 3 \times 7$
b) $225 = 3^2 \times 5^2$
c) $180 = 2^2 \times 3^2 \times 5$
d) $490 = 2 \times 5 \times 7^2$
e) $640 = 2^7 \times 5$
f) $243 = 3^5$

8. Compare your answers from question 4 to those in question 7.

The numbers in question 7 are multiples of the numbers in question 5. Using this information it is easy to determine the prime factorisation of the larger numbers. Example: $60 = 2^2 \times 3 \times 5$, but $180 = 60 \times 3$, therefore $180 = 2^2 \times 3 \times 5 \times 3$ or $2^2 \times 3^2 \times 5$.

9. Use your answers to question 7 to help write down the prime factorisation of each of the following:

a) $294 = 42 \times 7$
$= 2 \times 3 \times 7^2$

b) $4725 = 225 \times 21$
$= 3^3 \times 5^2 \times 7$

c) $2520 = 180 \times 14$
$= 2^3 \times 3^2 \times 5 \times 7$

d) $16170 = 490 \times 33$
$= 2 \times 3 \times 5 \times 7^2 \times 11$

e) $14080 = 640 \times 22$
$= 2^8 \times 5 \times 11$

f) $18711 = 234 \times 77$
$= 3^5 \times 7 \times 11$