# Compositions Graphically – ID: 9990

By Holly Thompson

Time required
*45 minutes*

## Activity Overview

*Students will use graphs and tables to find compositions of functions. Two of the compositions presented in this activity represent real-world situations, which should aid in students understanding the concept of compositions.*

## Concepts

- *Representing compositions in function notation*
- *Solving compositions graphically and using tables*

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## Teacher Preparation

*This investigation offers an approach to finding compositions by using a graph of each of the functions to be composed. This activity also helps students understand what types of situations can be represented by compositions, which in turn helps reinforce the mathematical definition of a composition.*

- *This activity could be used in Precalculus as a review on compositions from Algebra 2 or it can also be used as an introduction to compositions. It could also be used in an Algebra 2 class depending on your curriculum.*
- *Prior to beginning this activity, students should have a complete understanding of function notation.*
- *The screenshots on pages 2–4 (top) demonstrate expected student results. Refer to the screenshots on pages 4 (bottom) and 5 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "9990" in the quick search box.***

## Classroom Management

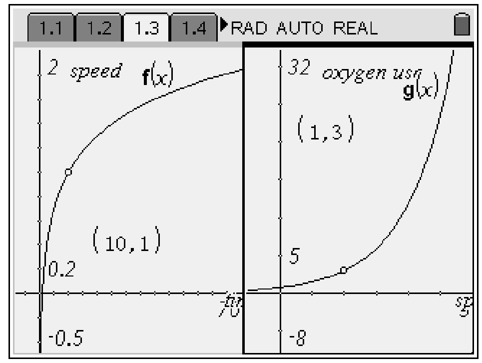
- *This activity is intended to be initially **teacher-led**, after which students may be able to work individually or with a partner. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds. Be sure to cover all the material necessary for students' total comprehension.*
- *The student worksheet PreCalcAct07_CompositionGraph_worksheet_EN is intended to guide students through the main ideas of the activity, and it also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*

## TI-Nspire™ Applications

*Graphs & Geometry, Lists & Spreadsheet, Notes*

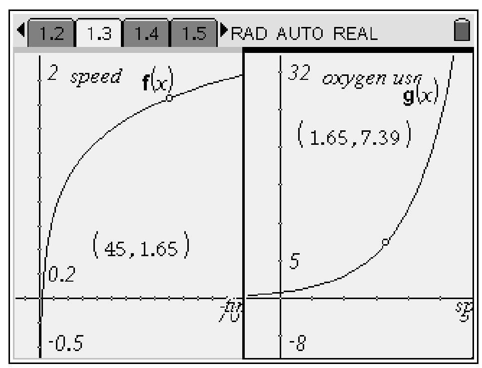
**Problem 1 – Swimming**




In this problem, students are given two graphs: **f**(*x*) represents a swimmer's speed in m/s in terms of time in seconds, and **g**(*x*) represents the same swimmer's oxygen use in L/min based on the speed in m/s. Students can drag the point on each of the graphs to investigate.

Students will unknowingly find a composition of these functions by answering the questions from page 1.4. First, they should move the point on **f**(*x*) to (10, 1). You should encourage students to talk about the *meaning* of this point—that is, at 10 seconds, the swimmer's speed is 1 m/s. Next, on the graph of **g**(*x*), students should locate the point (1, 3) and discuss what it means—when the swimmer's speed is 1 m/s, she is using 3 L/min of oxygen.

When they get to the third question, they should easily see that at 10 seconds, the swimmer is using 3 L/min of oxygen. You can use this as a springboard to either review or discuss the concept of a composition.



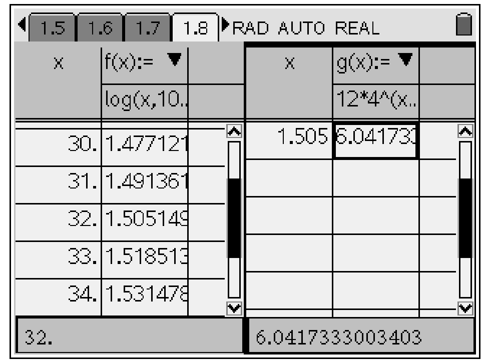

Students can then practice by answering the questions from pages 1.5 and 1.6 either individually or with a partner.

For the questions on page 1.6, students will need to "work backwards": beginning with a *y*-value and using it to find the *x*-value. This can easily be accomplished using their TI-Nspire handhelds by double-clicking and editing the coordinates of the point on each graph.

***Solutions***

- At 45 seconds, the swimmer is using 7.39 L/min of oxygen. (shown to the right)
- At 12 seconds, the swimmer is using 3.35 L/min of oxygen.
- If the swimmer is using 5 L/min of oxygen, she has been swimming for 23.44 seconds.
- If the swimmer is using 9 L/min of oxygen, she has been swimming for 61.66 seconds.

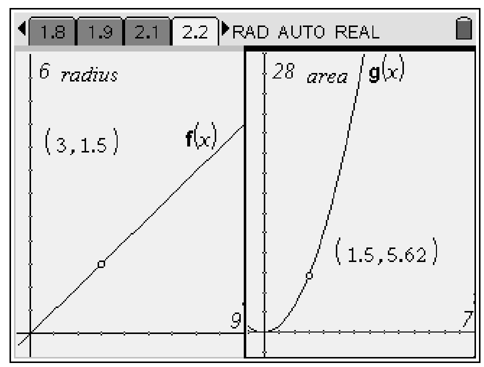



Page 1.7 asks students to use tables to find the compositions. The screenshot to the right shows how to use the function tables on page 1.8 to find the composition **g**(**f**(32)). After finding **f**(32) in the table on the left, they can either change the table step in the table on the right or set it up so as to be prompted for *x*-values.

Working the other way for the second question, students should find that a swimmer using 12 L/min of oxygen has been swimming for 100 seconds.

It is also important that students be able to define a composition of functions in their own words. One possible definition is: a way of combining two functions where the output of one function is used as the input for the other function.

**Problem 2 – An Oil Spill**




This problem provides a chance for students to practice with the compositions and to be introduced to the function notation that is used for compositions. The graph of **f**(*x*) is the radius of an oil spill in feet over the time in seconds. The graph of **g**(*x*) is the area of the same oil spill in square feet based on the radius in feet.

You may wish to work through the questions on page 2.3 with students so that you can discuss the notation for the composition.

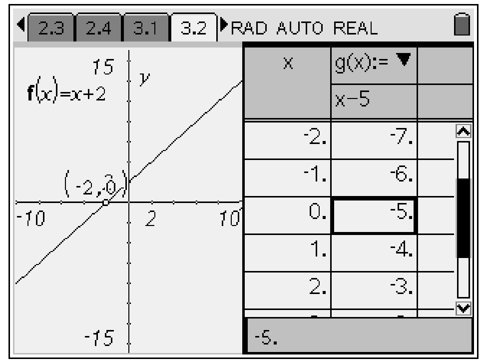
***Solutions***

- The area of the oil spill is 5.63 $ft^2$ after 3 seconds, i.e., **g**(**f**(3)) = 5.62 (shown above)
- The area of the oil spill is 16.9 $ft^2$ after 5.2 seconds, i.e., **g**(**f**(5.2)) = 16.9
- It took 2 seconds for the oil spill to have an area of 2.5 square feet, i.e., **g**(**f**(2)) = 2.5
- It took 6.32 seconds for the oil spill to have an area of 25 square feet, i.e., **g**(**f**(6.32)) = 25

**Problem 3 – Additional Practice**

Problem 3 gives **f**($x$) as a graph and **g**($x$) as a table with no context. All questions are asked in function notation so students can practice interpreting the notation using a graph and a table.



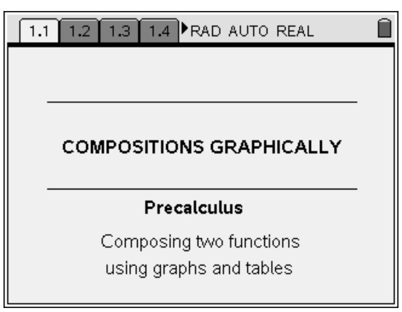

***Solutions***

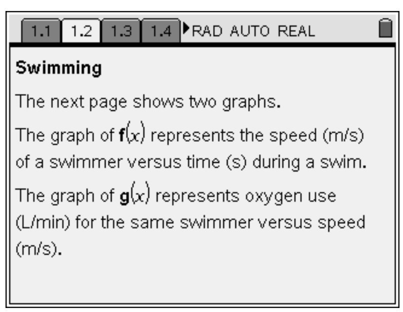
- **g**(**f**(–2)) = –5 (shown to the right)
- **f**(**g**(5)) = 2
- **g**(**f**($x$)) = –10, $x$ = –7
- **f**(**g**($x$)) = 3, $x$ = 6

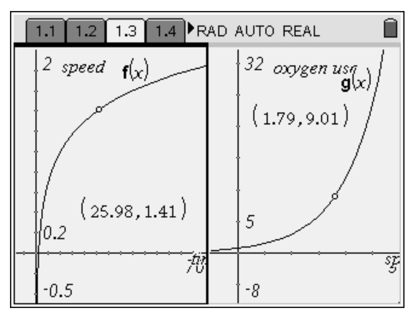
As a challenge, students are asked to determine the algebraic rules for **f**(**g**($x$)) and **g**(**f**($x$)). In this instance, they are both identical, and **f**(**g**($x$)) = **g**(**f**($x$)) = $x$ – 3.

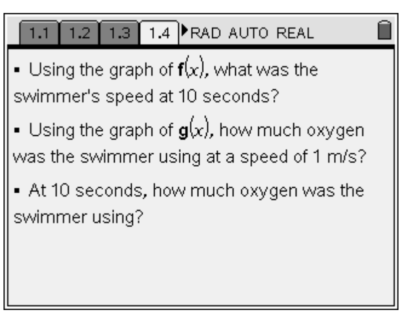
## Compositions Graphically – ID: 9990

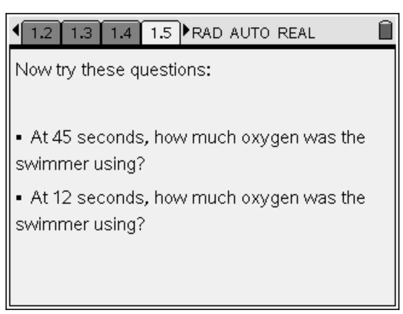
(Student)TI-Nspire File: *PreCalcAct07_CompositionGraph_EN.tns*

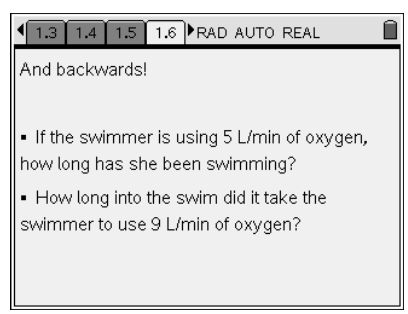


1.4 1.5 1.6 **1.7** RAD AUTO REAL

Now try it with tables. Use the function tables on the next page to answer these questions.

- A swimmer swimming for 32 seconds will use how much oxygen?
- A swimmer who is using 12 L/min of oxygen has been swimming for how long?

1.5 1.6 1.7 **1.8** RAD AUTO REAL

| x | f(x):= log(x,10.. |
|---|---|
| 0. | #UNDEF |
| 1. | 0. |
| 2. | .3010299 |
| 3. | .4771212 |
| 4. | .6020599 |

undef

| x | g(x):= 12*4^(x.. |
|---|---|
| 0. | .75 |
| 1. | 3. |
| 2. | 12. |
| 3. | 48. |
| 4. | 192. |

.75

1.6 1.7 1.8 **1.9** RAD AUTO REAL

- In your own words, how would you describe a composition of two functions?

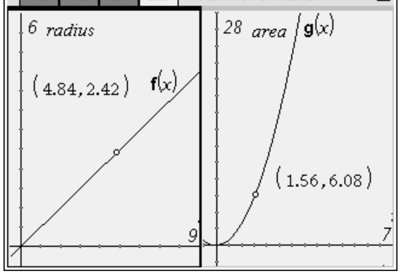
1.7 1.8 1.9 **2.1** RAD AUTO REAL

**An Oil Spill**

The next page shows the graphs of $f(x)$ and $g(x)$.

The graph of $f(x)$ shows the radius (feet) of an oil spill over time (s). The graph of $g(x)$ shows the area (square feet) of the oil spill based on the radius (feet).

1.8 1.9 2.1 **2.2** RAD AUTO REAL




1.9 2.1 2.2 **2.3** RAD AUTO REAL

- What is the area of the oil spill after 3 seconds? The way to represent this algebraically is $g(f(3))$.
- What is the area of the oil spill after 5.2 seconds—i.e., what is $g(f(5.2))$?

2.1 2.2 2.3 **2.4** RAD AUTO REAL

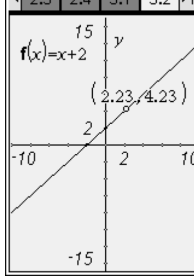
- How long did it take for the oil spill to have an area of 2.5 square feet? This can be represented algebraically as $g(f(x))=2.5$.
- Find $x$ if $g(f(x))=25$. Write what this means in words.

2.2 2.3 2.4 **3.1** RAD AUTO REAL

**Additional Practice**

The following page displays a graph for $f(x)$ and a table for $g(x)$. Find the compositions on page 3.3 using the graph and table.

2.3 2.4 3.1 **3.2** RAD AUTO REAL




| x | g(x):= x−5 |
|---|---|
| -2. | -7. |
| -1. | -6. |
| 0. | -5. |
| 1. | -4. |
| 2. | -3. |

-7.

2.4 3.1 3.2 **3.3** RAD AUTO REAL

Find the following:

- $g(f(-2)) = \square$
- $f(g(5)) = \square$
- $g(f(x)) = -10, \quad x = \square$
- $f(g(x)) = 3, \quad x = \square$

3.1 3.2 3.3 **3.4** RAD AUTO REAL

**A Challenge!**

- Can you find an algebraic expression for $f(g(x))$?
- How about $g(f(x))$?