EXTENDED LAW OF SINES

by Omar Hernández Rodríguez, EdD
University of Puerto Rico

INSTRUCTIONS FOR INSTRUCTORS

NCTM EXPECTATIONS
GRADES 9-12

- explore relationships (including congruence and similarity) among classes two- and three-dimensional geometry objects, make and test conjectures about them, and solve problems involving them;
- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;
- use trigonometric relationships to determine lengths and angle measures. (NCTM, 2000, p.397)

PUERTO RICAN INDICATORS For Grade 11

G.FG.11.5.5 Develop the Laws of Sine, the Law of Cosine and use them to determine lengths and angle measures in a triangle.
G.FG.11.6.1 Establish conjectures based on the exploration of geometric situations with or without the use of technology.
G.FG.11.6.2 Use direct or indirect proof to determine if a mathematical proposition is true.
G.FG.11.6.5 Organize and present direct and indirect proof using tables of two columns, paragraphs, and flux diagrams.

ACTIVITY OVERVIEW

This activity is to be used with pre-service mathematics teachers or in service teachers in a project of professional development. The main goal is to demonstrate how to use the TI-nspire technology to enhance the students capacity to elaborate and demonstrate conjectures. The elaboration of conjectures is fundamental in the development of the mathematical reasoning. With the information provided and the previous knowledge, the students form a judgment on the content that are studying Beginning in the early years of schooling, teachers can help students learn to make geometric conjectures. The National Council of Teachers of Mathematics (NCTM) recommends that teachers formulate questions similar to the following ones in the mathematical situations presented: What do you think will happen next? What is the pattern? Is this always true? What will happen if…? (NCTM, 2000, p57).

The relationship between the lengths of the sides of a triangle inscribed in a circle, the Sine of the internal angles and the radius of the circle is studied.

$$\frac{Sin\ A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = 2r$$

Where A, B, and C are the inner angles of the triangle ABC; a, b and c are the length of the sides of the triangle inscribed in a circle of center O and radius r. This property is called the Extended Law of Sines.

BEFORE THE ACTIVITY

Install the file Extended Law of Sines.tns in all the TI-handhelds. Students must have copy of the document LES-students.pdf or LES-students.doc. Students must be divided in groups of three.

DURING THE ACTIVITY

Students will discuss with their group mates if they have previously heard about the Extended Law of Sines. If it is so, they will try to remember the statement and will write it on the space that is provided.

All the students will turn to page that contains the statement of the Law. After moving the vertices of the triangle on the circumference and seeing the different representations of the Extended Law of Sines Law they will verify that it holds making the appropriate measurements as indicated (page 8). The demonstration of the Law is based on two theorems that have been seen previously:

a. The angle inscribed in a semicircle is a right angle.
b. Given B and C, two fixed points on a circle, for two points A and J on the circle $\measuredangle BAC \text{ and } \measuredangle BJC$ are congruent or supplementary.

The students will verify that these two properties always hold (page 11).

The proof of the Extended Law of the Sines begins on page 13. Students must describe thoroughly this page to the light of the two previous properties. Later, the students will give the reasons for the demonstration of the Law that is provided (page 14).

The professor must ask if this proof holds for all the cases and he (she) must observe that it is also necessary to proof that the Law holds in the case <BAC and <BJC are supplementary. Finally, the students will discover the relation between the center of the circle and circuncenter of the triangle.

In summary, the students will measure the angles, and the sides of the triangle, will discuss in group the questions proposed, will elaborate conjectures and they will verify them.

File to use: **Extended law of Sines.tns**.

AFTER THE ACTIVITY

Prove that:

1. the perpendicular bisectors of the sides of a triangle concur in a point .
2. the point of concurrence of the perpendicular bisectors of the sides of a triangle is the center of the circle circumscribed to the triangle.

SCREENS

1.1

The Extended Law of Sines

Omar Hernández

University of Puerto Rico

July 2008

1.2

This lesson is to be used with pre–service teachers learning how to use the TI–nspire handheld.

1.3

Question

Have you heard about the Extended Law of Sines?

Discuss with your group.

Answer

1.4

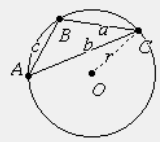
The Extended Law of Sines

For a triangle ABC inscribed in a circle of center O and radius r, the following is true

$$\frac{a}{SinA}=\frac{b}{SinB}=\frac{c}{SinC}=2r$$

1.5

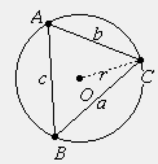
The Extended Law of Sines



Graphical representation

1.6

The Extended Law of Sines



$$\frac{a}{SinA}=\frac{b}{SinB}=\frac{c}{SinC}=2r$$

Graphical and *algebraic representations.*

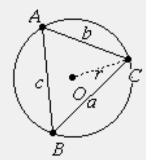
1.7

Exercise

On the next page measure angle A, the radius of the circle and the length of side a, and use *calculate* to verify that the Extended Law of Sines holds for angle A and side a.

1.8

The Extended Law of Sines 1 cm



$\measuredangle A=$

$x=a=$

$r=$

$\frac{x}{\sin(A)}=$

$2r=$

1.9

The proof of the Extended Law of Sines is based in two facts:

Fact 1: The angle inscribed in a semicircle is always a right angle.

Fact 2: Given B and C, two fixed points on a circle, for any two points A and J on the circle, it is true that are ∠BAC and ∠BJC are congruent or supplementary.

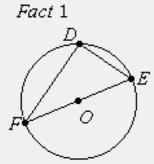
1.10

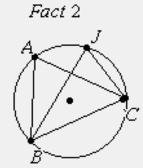
On the next page, measure ∠FDE, ∠BAC and ∠BJC .

Move the points on the circle to verify that the two facts on the previous page really hold.

1.11

Fact 1 *Fact 2* 1 cm





D =

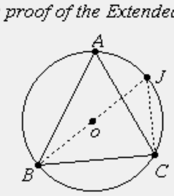
A =

J =

Slide points D, A and J & see what happens

1.12

The proof of the Extended Law of Sines



Describe the figure

1.13

Exercise

On the next page

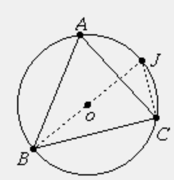
Give the reason for the proof of the Extended Law of Sines

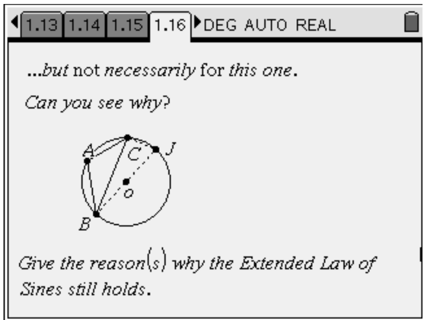
1.14

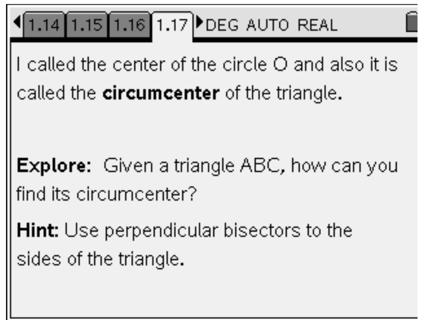
Statements	Reasons
$\sin J=\frac{a}{BJ}$	
$\sin J=\frac{a}{2r}$	
$\frac{a}{sinA}=\frac{a}{sinJ}=2r$	

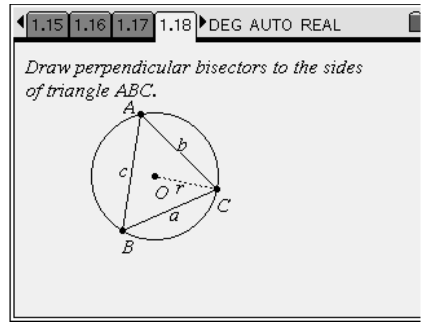
1.15

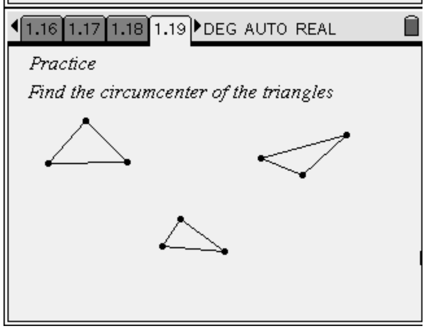
The proof worked for this triangle ...

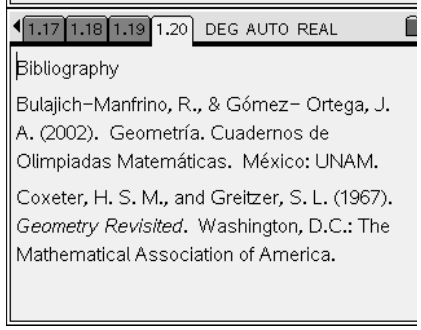












REFERENCES

Bulajich-Manfrino, R., & Gómez- Ortega, J. A. (2002). Geometría: Cuadernos de Olimpiadas Matemáticas. México: UNAM.

Coxeter, H. S. M., and Greitzer, S. L. (1967). Geometry Revisited. Washington, D.C.: The Mathematical Association of America.

National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA: Author.