Pick's Theorem

5615

Introduction

In this activity, students will establish Pick's Theorem, which is used to find the area of a polygon given on a grid.

Grades 9-12

NCTM Algebra Standards

- Represent and analyze mathematical situations and structures using algebraic symbols
- Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations

Files/Materials Needed

picks.act

1

a. Instruct your students to complete the four tables for the polygons shown on the activity sheet. Each group member should complete one of the tables then share their results.
b. Each student is to place only the data they collect into L1, L2, and L3 on their calculator.
c. Use **Screen Capture** to check student understanding. The student calculators should display three lists with six rows of data.

2

a. Have students create a scatter plot of the area versus the number of border points.
b. Have students use their calculators to find the best fit model for their scatter plot.
c. Use **Screen Capture** to check for student understanding. The student calculators should have a linear relationship with a slope of 0.5 and a variety of *y*-intercepts.

3

a. Launch TI-Navigator™ on the computer and start the session.
b. Have each student log into NavNet on their calculator.

4

a. Load the activity settings file ***Picks.act*** into Activity Center.
b. Start the activity and select the **Graph** tab to see the graphs of the equations from the class.
c. Stop the activity and select the **Graph-Equation** tab to see the graphs and equations.
d. The graphs should be the same for each member of the same group. The graphs displayed will be parallel lines.
e. Identify each of the graphs and the slope for each graph from each of the groups.
Ask: *"What is the impact of adding a point to the border to the area of a polygon?"*
Answer: It increases the area by 0.5.
f. Identify each of the graphs and the *y*-intercept from each of the groups.
Ask: *"What is the impact of adding an interior point to the inside of a polygon to the area of a polygon?"*
Answer: It increases the area by 1.

Pick's Theorem

5

Establish Pick's Theorem with the following questions and answers.

The first group of polygons each has no interior points. Let ***A*** be the area of a polygon, ***B*** the number of border points and ***I*** the number of interior points.

Ask: What function did you find for I = 0?

***A* = 0.5*B* – 1**

The second group of polygons each has one interior point.

What function did you find I = 1?

A* = 0.5*B

The third group of polygons each has two interior points.

What function did you find I = 2?

***A* = 0.5*B* + 1**

The fourth group of polygons each has three interior points.

What function did you find for I = 3?

***A* = 0.5*B* + 2**

What is the relationship between *I* and the *y*-intercept?

The number of interior points is one more than the *y*-intercept.

What is the function for ***A*** using both variables ***B*** and ***I*****?**

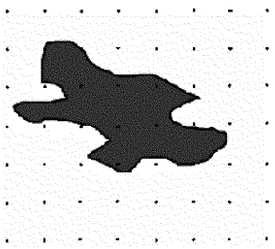
***A* = 0.5*B* + *I* – 1**

EXTENSION

6

a. Instruct your students to examine the graphic of the lake. Using the points of the grid for the endpoints of the sides of your polygon, draw a polygon around the lake's border. Count the number of border points and the number of interior points. Use the relationship we just found to compute the area on your calculator

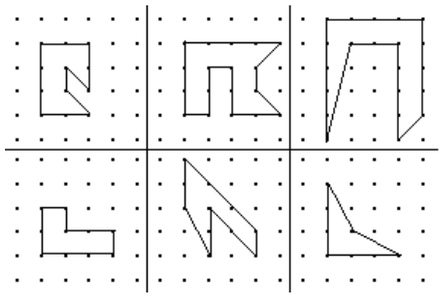
b. Estimate the size of this lake if the distance between the grid marks is 1 kilometer.



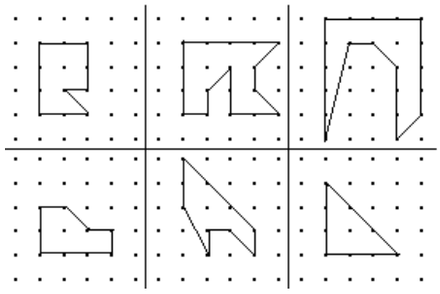
Pick's Theorem

Student Activity Sheet

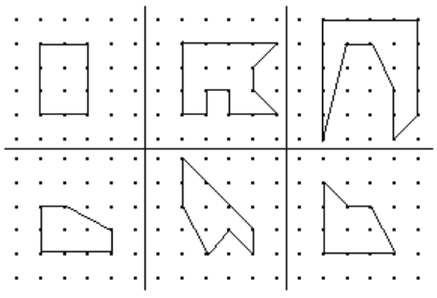
Find the area of the following polygons and count the number of border and interior points. Each group member should complete one table then share their results with the other group members.



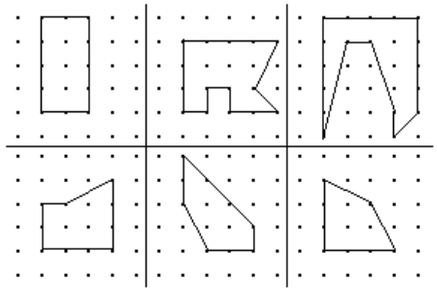
Area of each Polygon	Number of Border Points	Number of Interior Points



Area of each Polygon	Number of Border Points	Number of Interior Points



Area of each Polygon	Number of Border Points	Number of Interior Points



Area of each Polygon	Number of Border Points	Number of Interior Points