Grade level: 9-12
Subject: mathematics
Time required: 45 to 90 minutes

Exploration of the Trigonometric Identities using the Unit Circle

by – Christine Kasitz

Activity overview

Students will investigate the relationship of the trigonometric functions to similar triangles created using the unit circle.

Concepts

- Trigonometric Identities
- Similar Triangles
- Unit Circle
- Right Triangles
- Right Triangle Terminology
- Pythagorean Theorem
- Problem Solving
- Critical Thinking

Credits

This activity has been handed down through the generations. I am only taking the activity and applying it to new technology.

Teacher preparation

The teacher should work through the activity before presenting the activity in class. Also, students must be familiar with the trigonometric functions prior to the activity. The discussions during the activity should be completed together as a class.

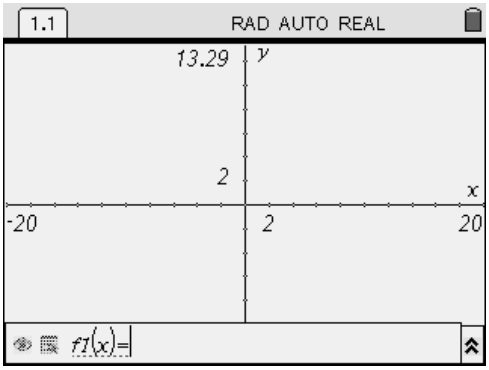
Classroom management tips

It is recommended that the students work with a partner during the activity with both students using calculators, if possible.

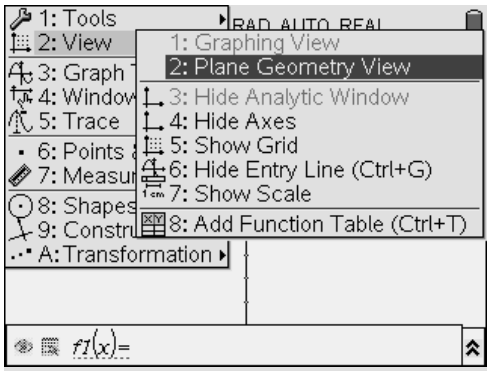
Step-by-step directions

Construct and Investigate

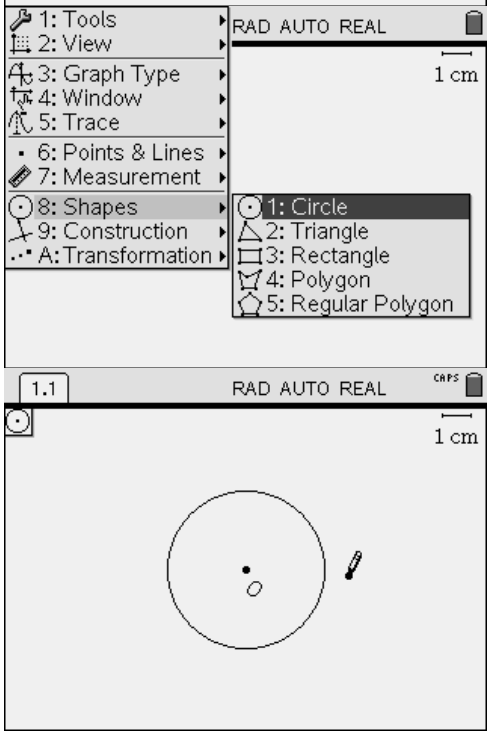
1) Open a new Graphs and Geometry Page



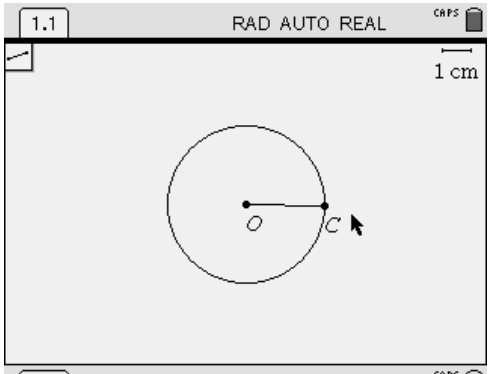
2) Hide the Axis
-Menu 2: View, 2: Plane Geometry View



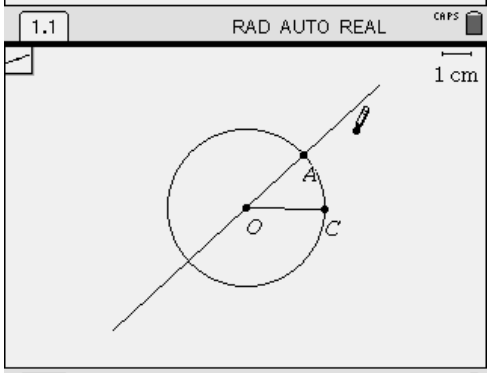
3) Construct a circle with center O.
- Menu 8: Shapes, 1: Circle



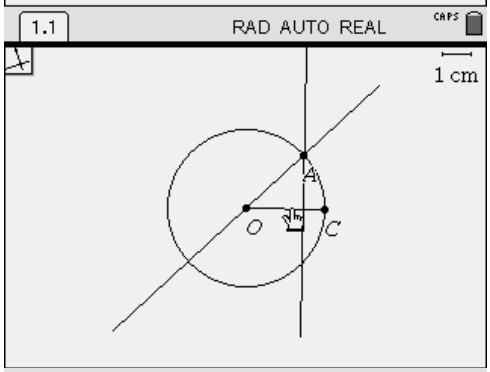
4) Draw a segment from the center, O, to the circle. Label the endpoint C.
- Menu, 6: Points and Line, 5: Segment.



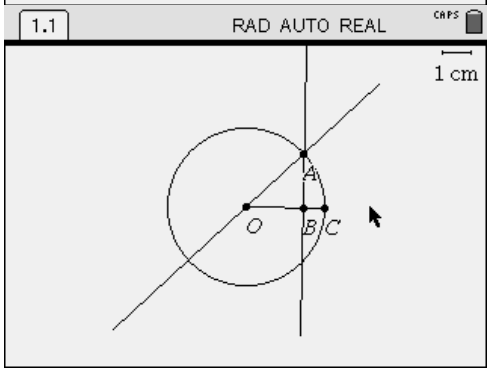
5) Draw a line through the center, O, of the circle with a point on the circle. Label the point, A.
- Menu, 6: Points and Lines, 4: Line



6) Construct a line perpendicular to the line segment OC through point A.
- Menu, 9: Constructions, 1: Perpendicular



7) Find and label the point of intersection, B.
- Menu, 6: Points and Lines, 3: Intersecting Points



8) Discussion: Analysis of the figure so far.

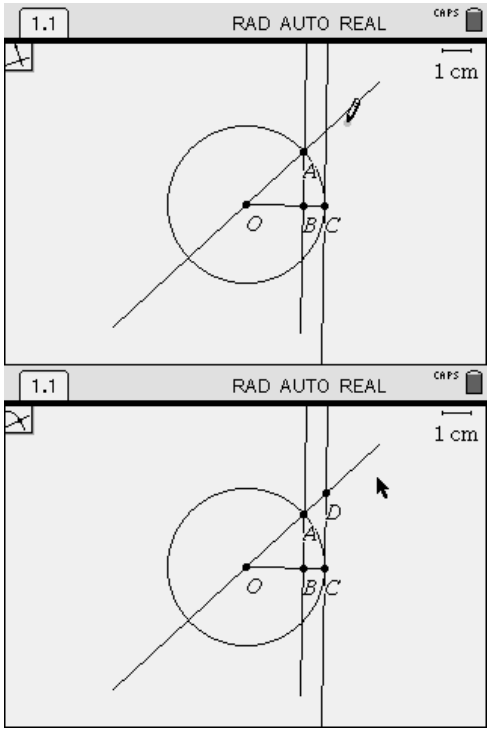
Given that $\overline{OC}=1$, it follows that $\overline{OA}=1$ since A is a point on the circle. Using the definition of sine and cosine to obtain the following:

$$\sin(BOA)=\frac{Opposite}{hypotenuse}=\frac{\overline{AB}}{1}=\overline{AB}$$

and

$$\cos(BOA)=\frac{Adjacent}{Hypotenuse}=\frac{\overline{OB}}{1}=\overline{OB}$$

9) To discover a relationship for the tangent function, construct a tangent line to the circle at point C.
-Menu, 9: Constructions, 1: Perpendicular



10) Label the point of intersection of the line and tangent line, D.

11) Discussion:
∠AOB ≈∠COD

$$\tan(BOA)=\frac{opposite}{hypotenuse}=\frac{\overline{AB}}{\overline{BO}}=\frac{\overline{CD}}{\overline{OA}}=\frac{\overline{CD}}{1}=\overline{CD}$$

12) Discussion:
What are the cofunctions?
How do are they related to sine, cosine, and tangent?

13) Discussion:

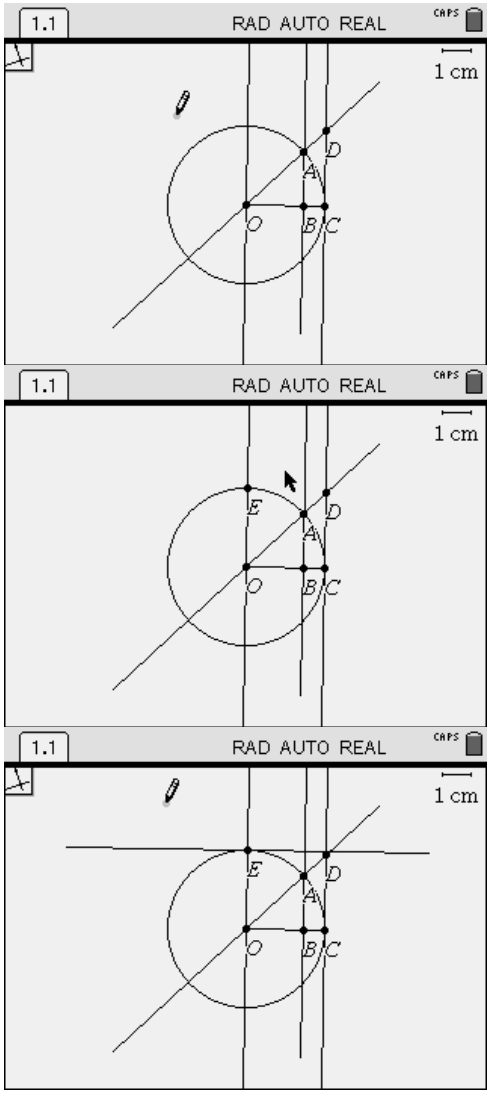
$$\sec(BOA) = \frac{1}{\sin(BOA)} = \frac{hypotenuse}{adjacent} = \frac{\overline{OD}}{\overline{OC}}$$

$$= \frac{\overline{OD}}{1} = \overline{OD}$$

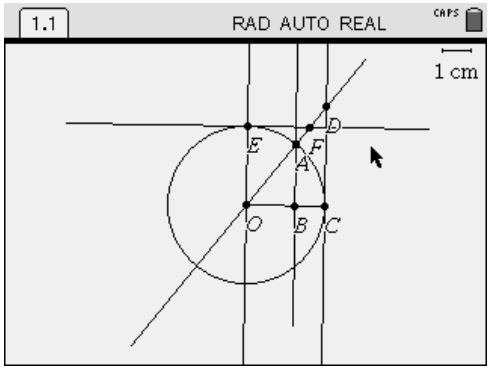
14) To find the cosecant, and cotangent functions, construct a line through O, perpendicular to OC.
-menu, 9: Construction, 1: Perpendicular

15) Find the point of intersection of the newly created line and the circle and label it E.
-menu, 6: Points and Lines, 3: Intersecting Points

16) Construct the tangent line at point E.
-menu, 9: Construction, 1: Perpendicular



17) Place a point at the intersection of the tangent line and line OD, label the point F.
-menu, 6: Points and Lines, 3: Intersecting Points



18) Discussion:

$\overline{EO} = \overline{OA} = \overline{OC} = 1$

∠AOB ≈∠OEF

$$\csc(BOA) = \frac{1}{\cos(BOA)} = \frac{hypotenuse}{adjacent} = \frac{\overline{OF}}{\overline{OE}}$$

$$= \frac{\overline{OF}}{1} = \overline{OF}$$

$$\cot(BOA) = \frac{1}{\tan(BOA)} = \frac{adjacent}{opposite} = \frac{\overline{EF}}{\overline{OE}}$$

$$= \frac{\overline{EF}}{1} = \overline{EF}$$

19) Worksheet – Developing the Pythagorean Identities

- Assessment and evaluation
- *Students should complete the student worksheet during the activity.*

Activity extensions

- *Students apply knowledge to textbook problems from the classroom textbook.*

Exploration of the Trigonometric Identities using the Unit Circle

by: Christine Kasitz

Grade level: secondary

Subject: mathematics

Time required: 45 to 90 minutes

Student TI-Nspire Document

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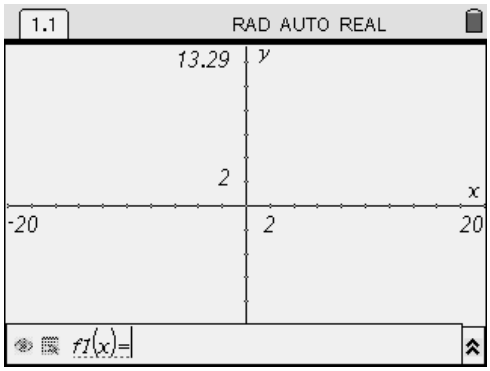
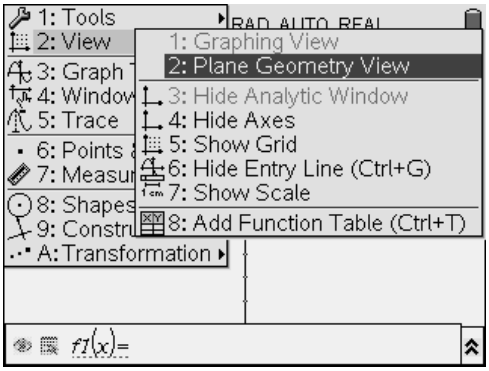
Screenshot #1



Screenshot #2



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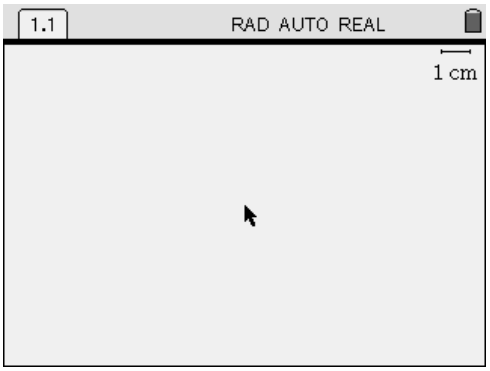

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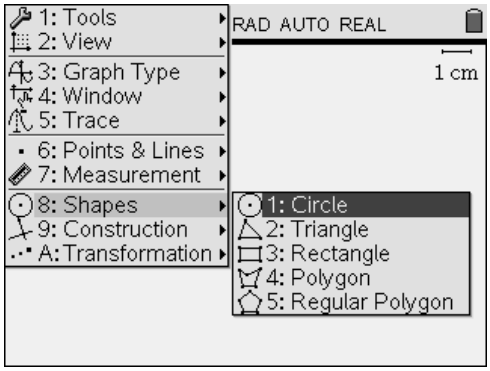

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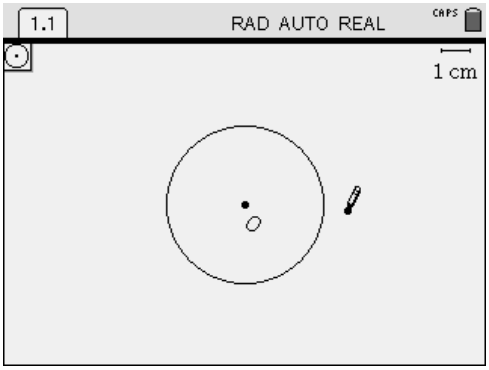

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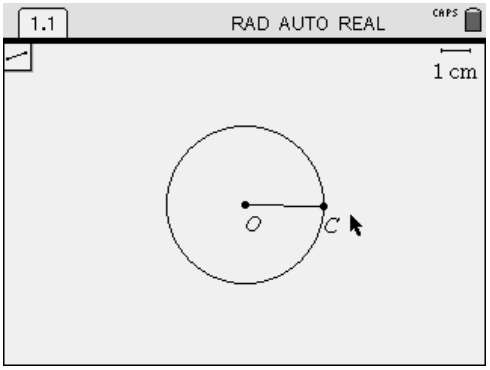

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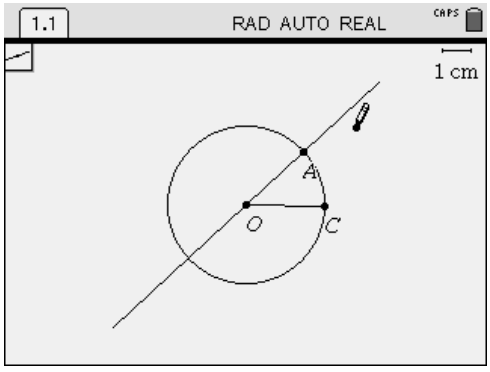

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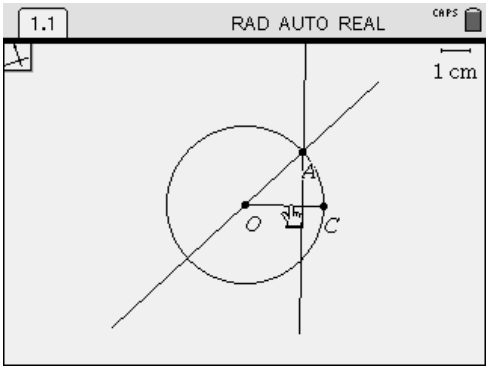

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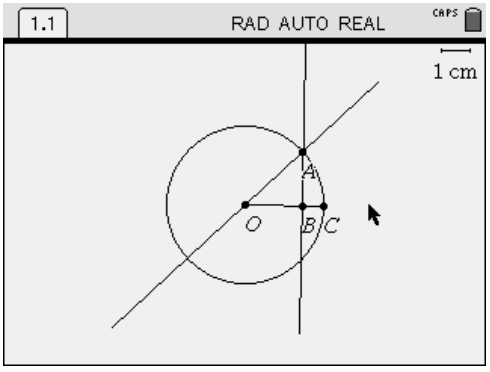

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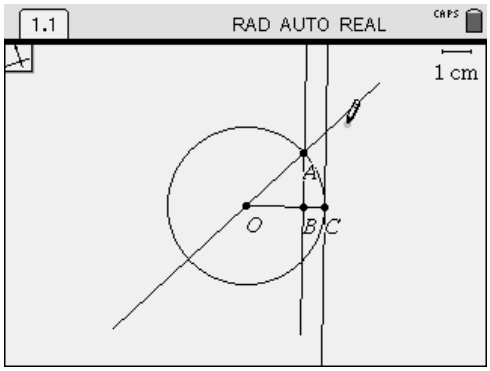

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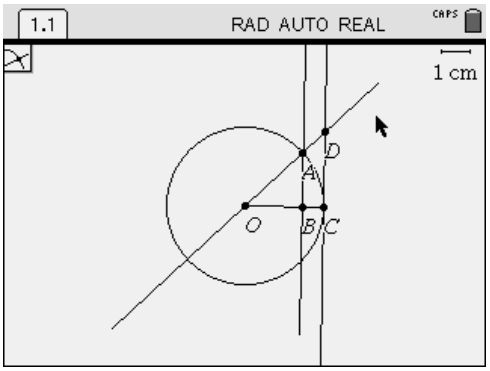

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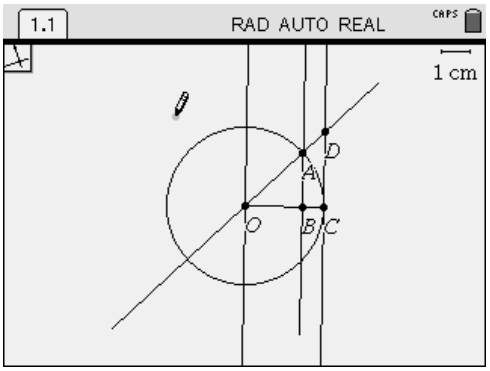

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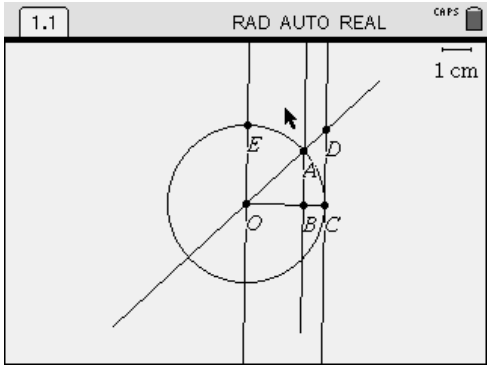

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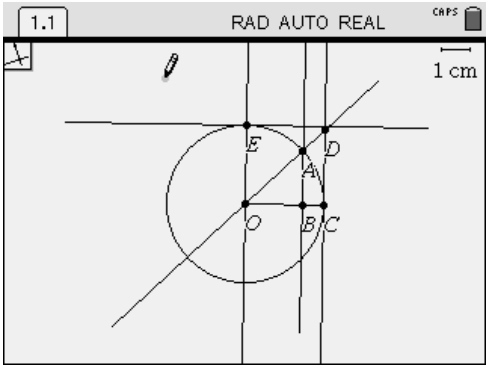

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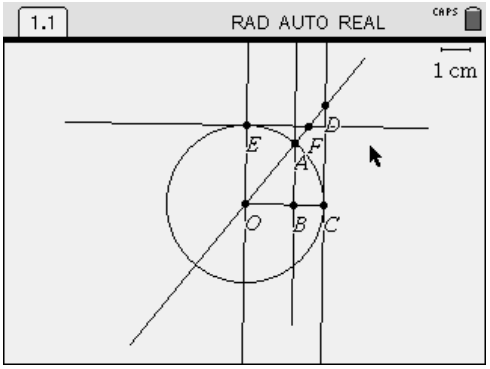

Exploration of the Trigonometric Identities using the Unit Circle Pythagorean Identities Worksheet

Name:_____

Class: _____

Answer the following equations based upon the results of the exploration.

1) Recreate the diagram from the Nspire in the space below. Show and develop the Pythagorean identity for sine and cosine.

2) Recreate the diagram from the Nspire in the space below. Show and develop the Pythagorean identity for secant and tangent.

3) Recreate the diagram from the Nspire in the space below. Show and develop the Pythagorean identity for cosecant and cotangent.