

by - Christine Kasitz

Activity overview

Students will investigate the relationship of the trigonometric functions to similar triangles created using the unit circle.

Concepts

- Trigonometric Identities
- Similar Triangles
- Unit Circle
- Right Triangles
- Right Triangle Terminology
- Pythagorean Theorem
- Problem Solving
- Critical Thinking

Credits

This activity has been handed down through the generations. I am only taking the activity and applying it to new technology.

Teacher preparation

The teacher should work through the activity before presenting the activity in class. Also, students must be familiar with the trigonometric functions prior to the activity. The discussions during the activity should be completed together as a class.

Classroom management tips

It is recommended that the students work with a partner during the activity with both students using calculators, if possible.

Step-by-step directions

Construct and Investigate

1) Open a new Graphs and Geometry Page





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2) Hide the Axis		
-Menu 2:	View, 2: Plane	Geometry View

3) Construct a circle with center O. - Menu 8: Shapes, 1: Circle

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RAD AUTO REAL

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CAPS

4) Draw a segment from the center, O, to the circle. Label the endpoint C.

- Menu, 6: Points and Line, 5: Segment.



1.1

6) Construct a line perpendicular to the line segment OC through point A.

Points

- Menu, 9: Constructions, 1: Perpendicular



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8) Discussion: Analysis of the figure so far.

Given that $\overline{OC}=1$, it follows that $\overline{OA}=1$ since A is a point on the circle. Using the definition of sine and cosine to obtain the following:

$$\sin(BOA) = \frac{Opposite}{hypotenuse} = \frac{\overline{AB}}{1} = \overline{AB}$$

and

$$\cos(BOA) = \frac{Adjacent}{Hypotenuse} = \frac{OB}{1} = \overline{OB}$$

9) To discover a relationship for the tangent function, construct a tangent line to the circle at point C. -Menu, 9: Constructions, 1: Perpendicular



10) Label the point of intersection of the line and tangent line, D.

11) Discussion: $\angle AOB \approx \angle COD$ $\tan(BOA) = \frac{opposite}{hypotenuse} = \frac{\overline{AB}}{\overline{BO}} = \frac{\overline{CD}}{\overline{OA}} = \frac{\overline{CD}}{1} = \overline{CD}$



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12) Discussion:What are the cofunctions?How do are they related to sine, cosine, and tangent?

13) Discussion:

$$\sec(BOA) = \frac{1}{\sin(BOA)} = \frac{hypotenuse}{adjacent} = \frac{\overline{OD}}{\overline{OC}}$$
$$= \frac{\overline{OD}}{1} = \overline{OD}$$

14) To find the cosecant, and cotangent functions, construct a line through O, perpendicular to OC. -menu, 9: Construction, 1: Perpendicular



16) Construct the tangent line at point E. -menu, 9: Construction, 1: Perpendicular





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17) Place a point at the intersection of the tangent line and line OD, label the point F. -menu, 6: Points and Lines, 3: Intersecting Points



18) Discussion: $\overline{EO} = \overline{OA} = \overline{OC} = 1$ $\angle AOB \approx \angle OEF$ $\csc(BOA) = \frac{1}{\cos(BOA)} = \frac{hypotenuse}{adjacent} = \frac{\overline{OF}}{\overline{OE}}$ $= \frac{\overline{OF}}{1} = \overline{OF}$ $\cot(BOA) = \frac{1}{\tan(BOA)} = \frac{adjacent}{opposite} = \frac{\overline{EF}}{\overline{OE}}$

$$=\frac{\overline{EF}}{1}=\overline{EF}$$

19) Worksheet – Developing the Pythagorean Identities

Assessment and evaluation

• Students should complete the student worksheet during the activity.

Activity extensions

• Students apply knowledge to textbook problems from the classroom textbook.



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Student TI-Nspire Document





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Exploration of the Trigonometric Identities using the Unit Circle Pythagorean Identities Worksheet

Name:_____

Class: _____

Answer the following equations based upon the results of the exploration.

1) Recreate the diagram from the Nspire in the space below. Show and develop the Pythagorean identity for sine and cosine.

2) Recreate the diagram from the Nspire in the space below. Show and develop the Pythagorean identity for secant and tangent.

3) Recreate the diagram from the Nspire in the space below. Show and develop the Pythagorean identity for cosecant and cotangent.