

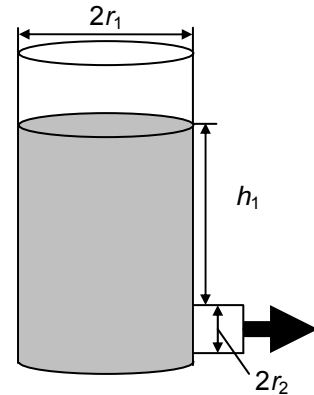


Torricelli's Law describes the relationship between the velocity of fluid leaving a container under the force of gravity and the height of the fluid. In its simplest form, Torricelli's law states that water in a tank will escape through a (small) hole in the bottom with the same speed that it would attain falling from the same height to the level of the hole. Torricelli's result may be calculated using the conservation of energy methods.

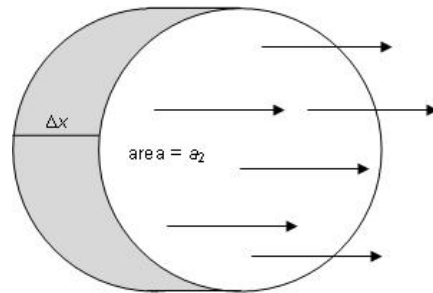
We want to find out if the following factors have an effect on the water flow: the initial height of the water above the tap, the height of the tap, the width of the tap.

Torricelli's Law states that  $v = \sqrt{2gh}$ , where  $v$  is the velocity,  $g$  is the acceleration due to gravity, and  $h$  is the height of the fluid in the container.

Consider the tank filled with water shown to the right. Let  $h_1$  be the height of the water above the center of the tap,  $r_1$  be the radius of the cylinder, and  $r_2$  be the radius of the tap. Let us examine this situation closer to see the rate in which the height of the water is decreasing. In this situation, we have  $v = \sqrt{2gh_1}$ .



Let's consider how water exits the tank. In looking closer at the tap, we can find the volume of a small cross-section. From this information, we get  $dV = a_2 \cdot \Delta x$ , where  $a_2$  is the area of the cross-section of the tap. The rate at which the volume changes with respect to time is  $\frac{dV}{dt} = a_2 \frac{dx}{dt}$  or  $\frac{dV}{dt} = a_2 v$ , since velocity is the first derivative of displacement. Therefore, the total volume in the tank ( $V$ ) decreases at a rate of  $a_2 v$ , or  $\frac{dV}{dt} = -a_2 v$ .



**Step 1:** Use the chain rule to write an expression for the rate of change of the volume with respect to time,  $\frac{dV}{dt}$ , in terms of  $\frac{dh}{dt}$ .

**Step 2:** Use what you know about the volume and height of a cylinder to create an expression for  $\frac{dV}{dh}$ . Substitute this into your equation from Step 1 and simplify.



**Step 3:** Find an expression for  $\frac{dh}{dt}$  in terms of  $v$ , the exit velocity of the water. (Hint: You should now have two different equations that involve the rate of change of volume and velocity.)

**Step 4:** Use the formula for the area of a circle to rewrite the equation in Step 3 using  $r_1$  and  $r_2$  instead of  $a_1$  and  $a_2$ .

**Step 5:** Torricelli's Law states the relationship between  $v$  and  $h$ . Substitute this into your result from Step 4.

**Step 6:** Integrate your equation from Step 5.

**Step 7:** Find the value of the constant of integration using the initial conditions  $t = 0$  and  $h = h_1$ . Solve your equation for  $h$ .

Looking at the variables in your solution to Step 7, what factors affect the velocity of the water?

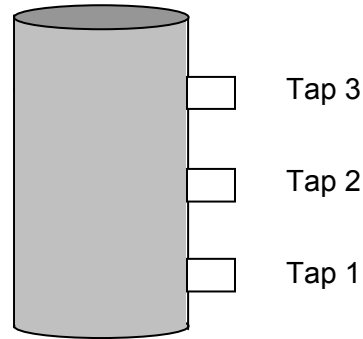


## Extension

Use principles of projectile motion to derive the trajectory equation for the flow of water. The picture at the right is not drawn to scale. For each tap, let  $h_1$  be the height from the top of the water to the center of that tap. Assume that water is only flowing from one tap at a time and that the container is refilled before changing taps.

a. State initial conditions.

b. Derive the horizontal equations of motion.



c. Derive the vertical equations of motion.

d. Combine horizontal and vertical components to form the trajectory equation.

e. Find the trajectory equations for the flow of water from each tap using the following information: radius of cylinder = 2 ft, radius of taps = 0.4 ft, initial height of the water = 10 ft.

Tap 1:  $h_1 = 8$  ft

Tap 2:  $h_1 = 5.5$  ft

Tap 3:  $h_1 = 2.7$  ft