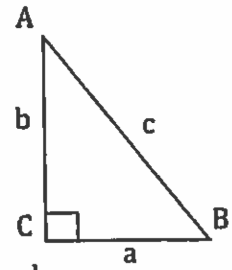
Nspire Activity: Déja Vu and the Law of Cosines

In this activity you will look at triangles where two sides and an included angle (SAS triangles) are given. You will derive a formula to determine missing sides and angles for these types of triangles.



1. State the Pythagorean Theorem relating the sides in the right triangle at right.

   $a^2 + b^2 = c^2$

2. Based on the Pythagorean Theorem, determine the value of $a^2 + b^2 - c^2$ for the right triangle.

   0

3. Suppose sides 'a' and 'b' stay constant and angle C becomes an acute angle. Predict whether the value of '$a^2 + b^2 - c^2$' will be positive, negative, or zero. Explain.

   If 'a' & 'b' are constant, a^2+b^2 is constant; If angle C is acute, 'c' will be shorter, thus making $a^2+b^2-c^2 > 0$.

4. Suppose sides 'a' and 'b' stay constant and angle C becomes an obtuse angle. Predict whether the value of '$a^2 + b^2 - c^2$' will be positive, negative, or zero. Explain.

   If 'a' and 'b' are constant, a^2+b^2 is constant; if angle C is obtuse, 'c' will be longer, thus making $a^2+b^2-c^2 < 0$.

5. Open the document 'lawofcos' and move to page 1.2. You will see ΔABC with side lengths calculated. The measurements for segment 'a' (= 3) and segment 'b' (=5) have been locked so they will not change. The value for $a^2 + b^2 - c^2$ has also been calculated. Verify your predictions from #3 and #4 by dragging on vertices of ΔABC to make angle 'C' acute and obtuse.

6. Continue dragging on a vertex of ΔABC and observe the range of values that $a^2 + b^2 - c^2$ takes on for all possible measures of angle C. Write the set of values observed in interval notation below.

   $(-30, 30)$

7. Move to page 2.1. You will see the same triangle and measurements from the previous page. There is also a graph below. You will observe the scatterplot that is formed when 'x' is represented by the measure of angle C and 'y' is represented by the value $a^2 + b^2 - c^2$. (*You should see a point currently plotted at (90°, 0) to represent 'x' and 'y' when angle C is equal to 90°*).

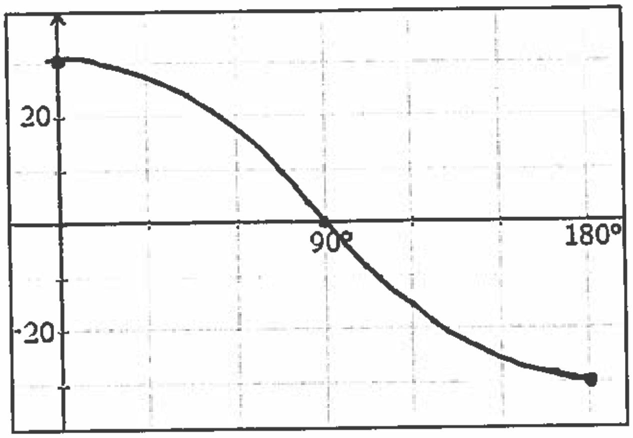
8. Grab point A and slowly drag the point so that angle C takes on a range of measurements in the interval (0°, 180°). Observe the graph formed at the bottom of the page.

   What type of function would best fit this data? Explain your choice and state properties of the graph that support your choice.

   A cosine function models this data; the graph is $y = 30\cos x$ from $[0°, 180°]$

9. Make a sketch of the graph you see at the bottom of page 2.1 below. Then, write an appropriate trig function that best fits this data.

Sketch:



Function:

$y = 30 \cos x$

10. Using your function written above, set up an equation by substituting expressions for 'x' and 'y' based on what 'x' and 'y' represent. The variable definitions are re-written below.

'x' = measure of angle C 'y' = $a^2 + b^2 - c^2$

$a^2+b^2-c^2 = 30 \cos C$

11. At this point you should have one numeric coefficient remaining in your equation. Recall that 'a' = 3 and 'b' = 5. How does the <u>remaining coefficient</u> in your equation relate to the values for these two fixed sides?

$30 = 2 \cdot (3 \cdot 5)$, so $30 = 2ab$

12. Re-write the coefficient in your equation in terms of 'a' and 'b' and re-write the equation below.

$a^2+b^2-c^2 = 2ab \cos C$

13. Now, solve your equation in step #12 for c^2. This is the Law of Cosines!!

<u>Law of Cosines:</u> $c^2 = a^2+b^2-2ab\cos C$