

Solids of Revolution

ID: 17390

Time required
45 minutes

Activity Overview

Students will investigate 3D visualizations of volumes created by rotating a function about the x - or y -axis. They will understand the concept and reason for the volume formula in order to be prepared for generalizations. Students will solve the definite integral by hand using the fundamental theorem of calculus and using the definite integral capabilities of the TI-Nspire. They will compare their solution with the volume of a cone that would be the same as or approximate the shape.

Topic: Application of Integral

- Calculate the volume generated by rotating a function about the x - or y -axis.
- Set up the definite integral, identify the limits of integration, the radius, and dh .

Teacher Preparation & Notes

- This investigation offers an introduction to solids of revolution. Most AP problems extend what is learned in this activity to rotate about lines like $y=2$ instead $y=0$ or $x=0$. Questions are also asked about volumes formed by cross sections with various shapes. See the related activities for follow up investigations.
- This exploration can be done at the students' own pace or with guided instruction.
- Many of the pages can be modified and used for other functions. If this is done, changing the window (press **menu**) may be necessary.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity. Questions are in "Exam" mode. If you are not using a Navigator to automatically grade the activity, you may want to change them "Self-Check" so students can get immediate feedback. With the Teacher Software, on a Question page, like page 1.2, choose Teacher Tool Palette under the menu options. Then click "Question Properties ..." and change the Document Type in the pop-up dialogue box.
- **To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "17390" in the keyword search box.**

Associated Materials

- SolidsOfRevolution_Student.doc
- SolidsOfRevolution.tns

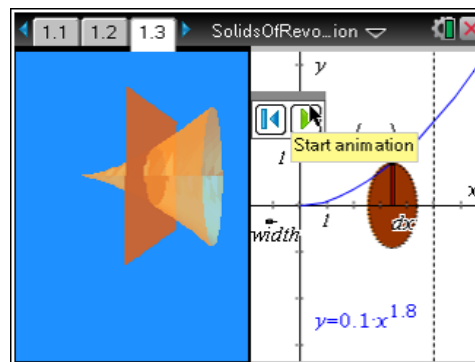
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Solids of Revolution - Disks (TI-Nspire technology) — 16105
- Visualizing Solids of Revolution - Washers (TI-Nspire technology) — 16106
- Volume by Cross Sections (TI-Nspire technology) — 12281

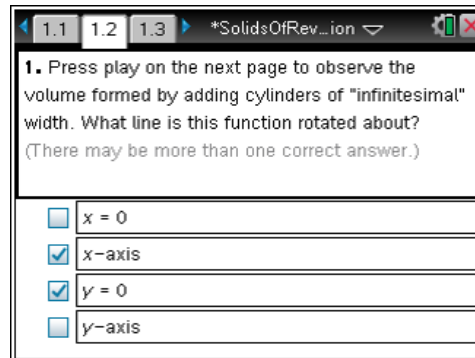
Tech Tips: When a 3D graph application is selected, the shape can be automatically rotated by pressing **A**. Press **esc** to stop. Press **menu** for other options, like zoom in and zoom out (press \times and \div).

- On the handheld, the animation may take a few moments to pause. Just click it once.
- The point dx can be grabbed (**ctrl click**) and manually moved.



Noting that the line the function is being rotated about is the line $y = 0$, can help pave the way for the next lesson that finds the volume between two functions. It can be said that is what is happening in this lesson, but one of the 'curves' is $y = 0$.

Marking the point (x, y) and drawing a thin slice are extremely helpful in identifying the important parts to solve for the volume.



1. The animation on page 1.3 shows the graph $y=0.1x^{1.8}$ with the point (x, y) and a thin slice. What is this function rotated about?

The function is rotated about the x-axis. This is the same thing as the line $y=0$

TI-Nspire Navigator Opportunity: Quick Poll, Live Presenter

See Note 1 at the end of this lesson.

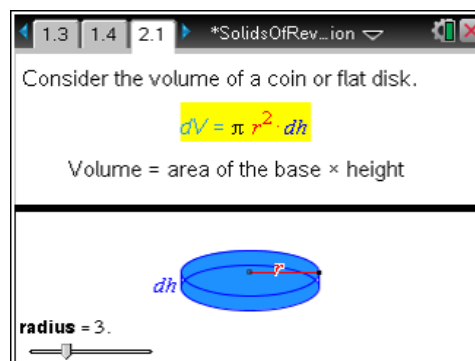
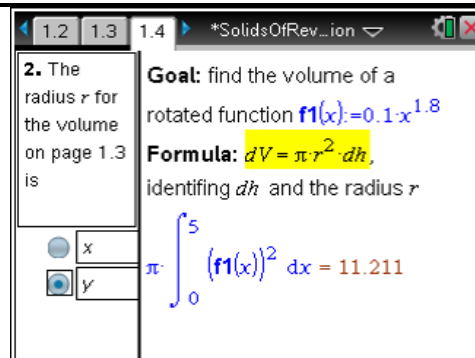
2. a. The thin slice helps identify the r in the equation $dV = \pi r^2 dh$. For the volume on page 1.3, what is the radius r ? y

The equation $dV = \pi r^2 dh$ can be explored on page 2.1. Instruct students to use the slider to change the radius and move the center of the circle.

- b. Draw a rotated thin slice and identify the parts. Explain the equation for an infinitesimal amount of volume $dV = \pi r^2 dh$.

Page 2.2 helps students understand the two possibilities for dh .

Quick Poll or discussion question: If the slab is drawn as on page 2.1, will the dh have an infinitesimal height of dx or dy ? Answer: The dh corresponds to dy . This will occur when the graph is rotated about the y -axis or $x=0$.



3. a. The area bounded by $x = 0$, $y = 0$ and the line $y = 10 - 2x$ is rotated about the x -axis. Find the volume. Show your work.

Solution:

$$dV = \pi \cdot \int_0^5 (10 - 2x)^2 dx = \frac{500\pi}{3}$$

$$\begin{aligned} dV &= \pi \cdot \int_0^5 (100 - 40x + 4x^2) dx \\ &= \pi \cdot \left(100 \cdot 5 - 20 \cdot 5^2 + \frac{4}{3} \cdot 5^3 \right) - \pi \cdot 0 \\ &= \frac{500\pi}{3} \end{aligned}$$

The volume of cone is $\frac{1}{3}\pi r^2 h$, where r is the radius and h is the height. Show that this agrees with the result you got from using the definite integral.

$$V = \frac{1}{3} \cdot \pi \cdot 10^2 \cdot 5 = \frac{500\pi}{3}$$

3. b. The area bounded by $x = 0$, $y = 0$ and the line $y = 10 - 2x$ is rotated about the y -axis. Find the volume. Show your work.

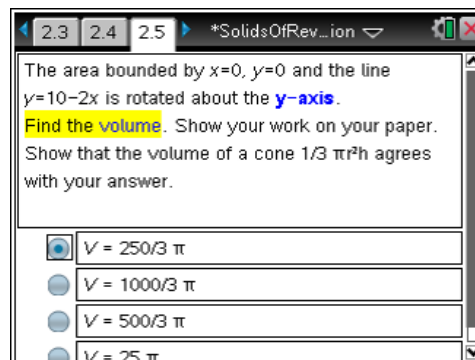
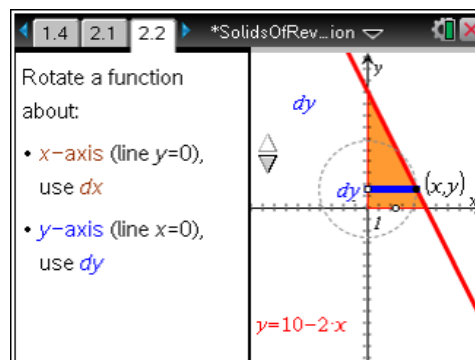
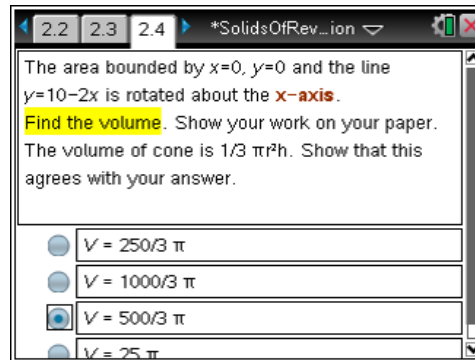
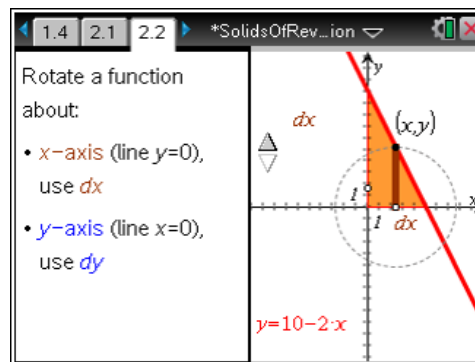
Solution:

$$dV = \pi \cdot \int_0^{10} \left(5 - \frac{1}{2}y \right)^2 dy = \frac{250\pi}{3}$$

$$\begin{aligned} dV &= \pi \cdot \int_0^{10} \left(25 - 5y + \frac{1}{4}y^2 \right) dy \\ &= \pi \cdot \left(25 \cdot 10 - \frac{5}{2} \cdot 10^2 + \frac{1}{12} \cdot 10^3 \right) - \pi \cdot 0 \\ &= \frac{250\pi}{3} \end{aligned}$$

The volume of cone is $\frac{1}{3}\pi r^2 h$, where r is the radius and h is the height. Show that this agrees with the result you got from using the definite integral.

$$V = \frac{1}{3} \cdot \pi \cdot 5^2 \cdot 10 = \frac{250\pi}{3}$$



4. A paraboloid can be formed by rotating the equation $y = x^2$ about the y -axis as shown on page 2.7.

Is the radius x or y for this paraboloid? x

Write the set up for the definite integral and apply the fundamental theorem to show your work.

Solution: See page 3.3 to the right.

This page uses the CAS functionality of solving for x , although this can easily be done by hand. This feature is utilized to all the previous page to become more of an exploration.

If the thin slice is dy , the limits of integration will be from $y = a$ to $y = b$. What is the volume of an inscribed cone of height of 4? How does this compare with the volume of the paraboloid you found? Explain. Conjecture what the formula for a paraboloid is.

$$V = \frac{1}{3} \cdot \pi \cdot 2^2 \cdot 4 = \frac{16\pi}{3}$$

$$V = \frac{1}{2} \cdot \pi \cdot 2^2 \cdot 4 = 8\pi$$

The volume of the circumscribed cylinder is 16π , so it can be deduced that the volume of the paraboloid is $\frac{1}{2}\pi r^2 h$.

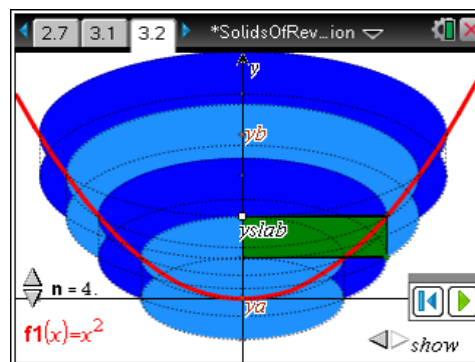
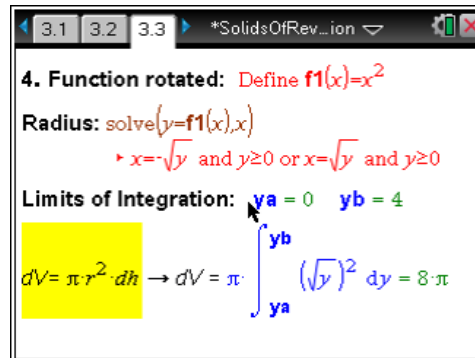
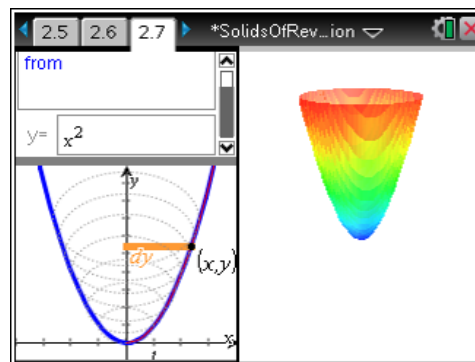
Quick Poll and/or discussion question: From page 3.2, using these slabs on this function, if Riemann sums were used to solve this problem would the answer (A) underestimate or (B) overestimate the exact answer?

Answer: Overestimate

See if students can write the Riemann sum in sigma notation with 4 subintervals.

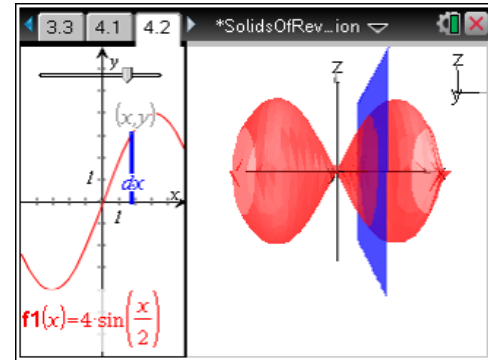
$$\pi \cdot \sum_{n=0}^4 \left(\sqrt{2}^2 \cdot \frac{4-0}{4} \right) = 10\pi$$

Challenge students to sketch a curve that would be rotated to give an underestimate.



5. Move to page 4.1. Find the volume to 3 decimal places for $f_1(x) = 4 \sin\left(\frac{x}{2}\right)$ from $x = -5$ to $x = 5$, rotated about the x -axis. Show the set up of the definite integral.

$$dV = \pi \cdot \int_{-5}^5 \left(4 \sin\left(\frac{x}{2}\right)\right)^2 dx \approx 299.528$$



TI-Nspire Navigator Opportunities

Note 1

Problem 1, *Quick Poll, Live Presenter*

Quick Polls can be used on any page that with a question, like 1.2, 1.4, 2.4, 2.5, and 2.7. It may be helpful to students if *Live Presenter* is used to discuss the interactivity of a 3D graph application. If the 3D graph application is selected, the arrows or A can be used to rotate the graph. This dynamic graph will help students visualize the problem. Also an extra quick poll can be asked to see how many students think they know the volume of a cone. Is the graph on page 1.3 a cone? No, but it can be approximated with a cone.

Note 2

Problem 2, *Collect, Review, Save to Portfolio*

After the activity is completed, collect the file and recap what was learned by reviewing students' answers in the review workspace. This formative assessment can be documented by saving the file to the portfolio where the grade will automatically be calculated.