Maximizing Area

by – Vicki Carter

Activity overview

Students are presented with a classic optimization problem of a rectangle with two vertices on the x-axis and two vertices on a parabola. They will explore the concept of maximum area of the rectangle graphically, numerically, and algebraically.

Concepts

Area of a geometric figure in the coordinate plane
Data collection – representation and interpretation using scatter plots
Derivatives and critical points

Teacher preparation

This investigation could be used as an introduction to optimization (max-min) problems in precalculus or calculus. Students should be familiar with polynomial function. They should also be able to represent coordinates in terms of functions.

- *Download the Maximizing Area.tns file.*

Classroom management tips

*This activity is intended to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Students will answer the questions posed on the Q&A Notes pages.*

- *As all questions are posed in the .tns file, the intent of this activity is for the teacher to collect the document from the students at the conclusion of the activity. As an alternative, you may wish to have the class record their answers on a separate sheet of paper or simply use the questions posed to engage the students in a class discussion.*

TI-Nspire Applications

Graphs & Geometry, Lists & Spreadsheet, Notes, Notes with Q&A templates, Calculator

TEXAS INSTRUMENTS

Maximizing Area

by: Vicki Carter

Grade level: 9-12
Subject: Precalculus, Calculus
Time required: 45 minutes

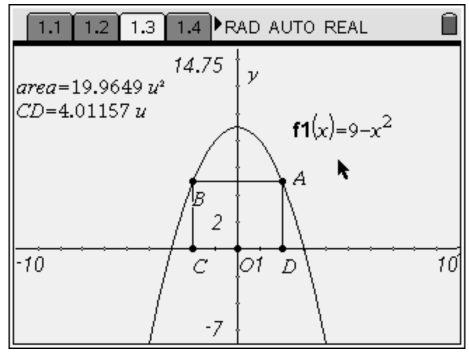
Materials: Maximizing Area.tns

Step-by-step directions

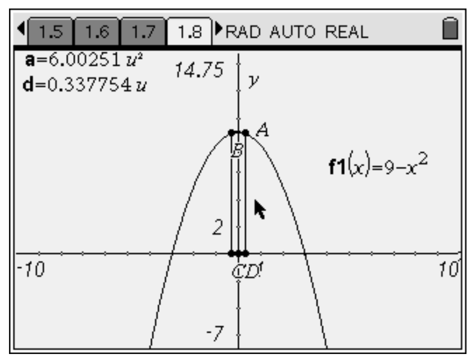
Investigating the Area of a Rectangle inscribed under a parabola graphically and numerically

Step 1: Students should grab point A and observe the changes in area of the rectangle and the length of side CD. After moving the point A to various positions in Quadrant I, they should be able to find the largest rectangle and answer the question on page 1.4. The question on page 1.6 is about the domain of the area function, $Ar(x) = 2x\left(9 - x^2\right)$.

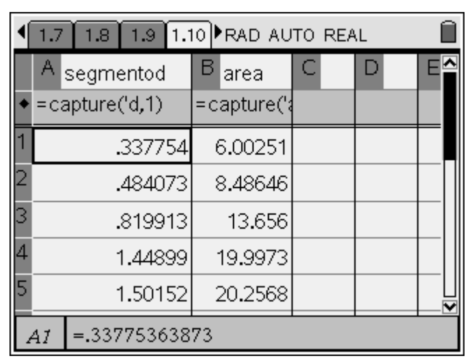
Students may need to go back to page 1.3 to observe that the length of side OD, or x, is between 0 and 3.



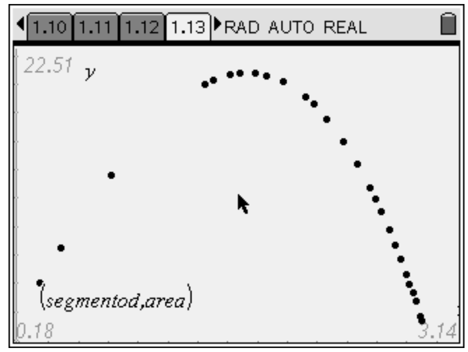
Step 2: Students are instructed to again drag point A along the parabola on page 1.8. The figure on page 1.8 starts with a small value for the length of OD, or **d**. Students should not try to find a smaller rectangle. They will move point A to increase the length of side OD, or **d**. Students should be instructed to keep point A within Quadrant I.



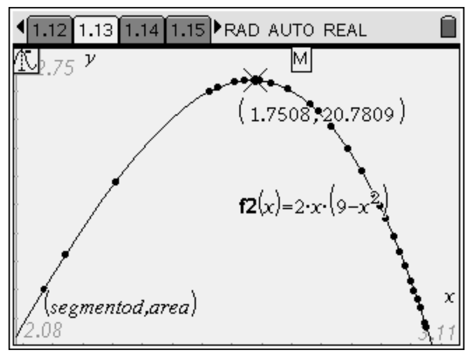
Step 3: As the students move point A, a table of values for the area and side OA are captured automatically. Here is an example of what their table should look like. Students are instructed to investigate the area column, column B, and answer the question on page 1.11.



Step 4: The resulting scatter plot is produced on page 1.13. Discuss the type of function that produces this graph. The student have already been given the area function earlier in the document, page 1.5, so they should be able to understand that this scatter plot is part of a cubic polynomial and not a parabola.



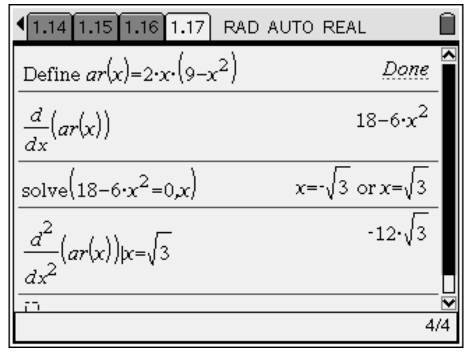
Step 5: On page 1.15, students are instructed to return to the scatter plot to graph the area function. Students will need to display the Entry Line. **MENU > 2:View > 6:Show Entry Line** or use **Ctrl+G**. The area function will be typed into *f2(x)*. The students should be instructed to trace on the graph. **MENU > 5: Trace > 1: Graph Trace.** Care should be taken to insure that the students are tracing on the function and not the scatter plot. Pressing up on the Navigation Pad should toggle them between the scatter plot and the function trace. They should trace on this function to find the maximum, M. You may want to have the students compare the maximum area recorded from the graphical investigation and from the table with this maximum value.

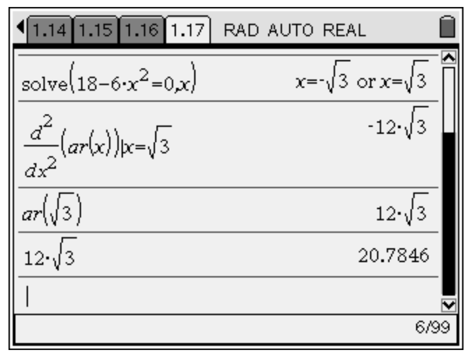


Extension: Investigating the Area of a Rectangle inscribed under a parabola algebraically
The second part (Problem 2) of this activity is intended to be ***teacher-led****. The use of the TI-Nspire™ CAS is necessary for this part of the activity. This is an investigation for calculus students.*

Step 6: On page 1.16, students are instructed to insert a Calculator application page to investigate the calculus part of this problem. **Home > 1: Calculator** With your assistance, students should discuss the mathematics they need to calculate a maximum value of a function. The recommended procedure is:

- **Menu > 1: Actions > 1: Define**
- Select the derivative template
- **Menu > 4: Algebra > 1: Solve** Find the zeros of the derivative
- Select the nth derivative template to evaluate the 2nd derivative of $Ar(x)$ at $x = \sqrt{3}$ to justify the maximum via the Second Derivative Test.
- Evaluate the area function at $x = \sqrt{3}$ to find the maximum area.





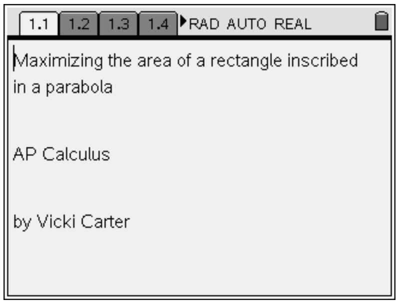
Step 7: You may want to have the students compare the maximum area recorded from the graphical investigation, from the table, and from the function graph with this maximum value.

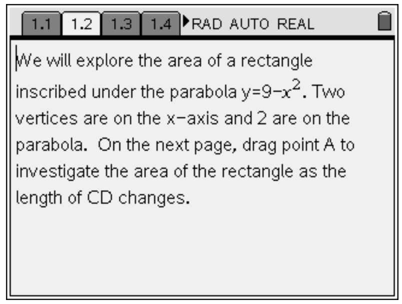
Assessment and evaluation

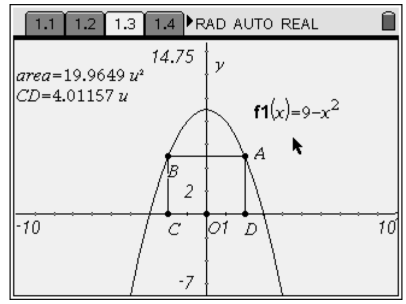
The teacher could collect the document from the students at the conclusion of the activity to check for understanding. As an alternative, you may wish to have the class record their answers on a separate sheet of paper or simply use the questions posed to engage the students in a class discussion.

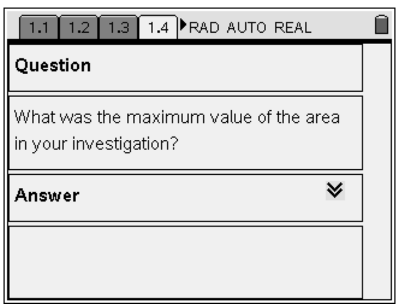
Student TI-Nspire Document

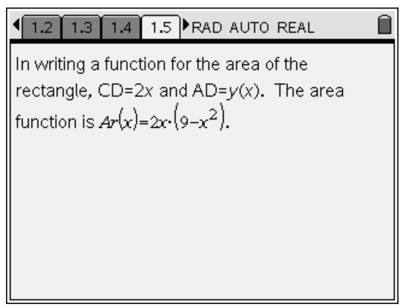
Maximizing Area.tns

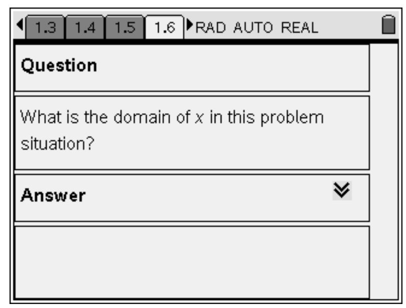


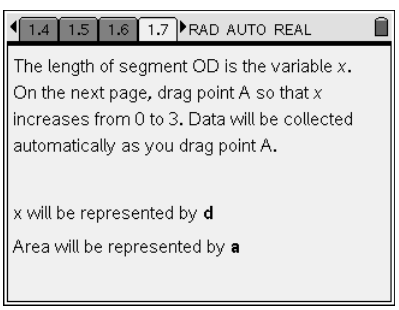


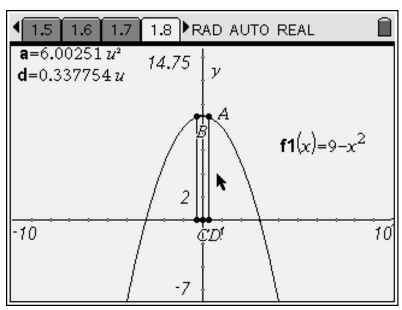


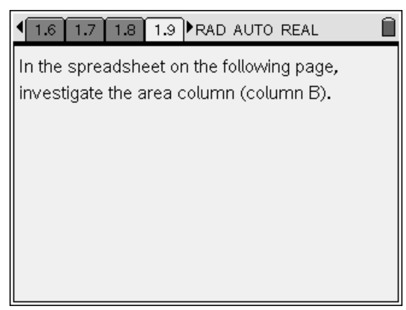


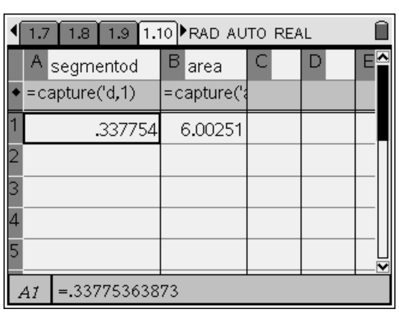


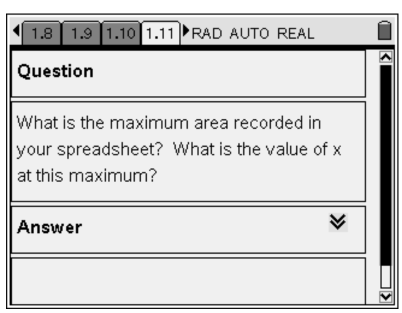


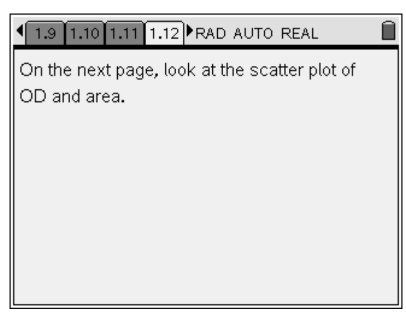


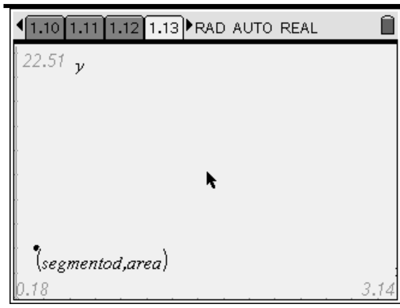








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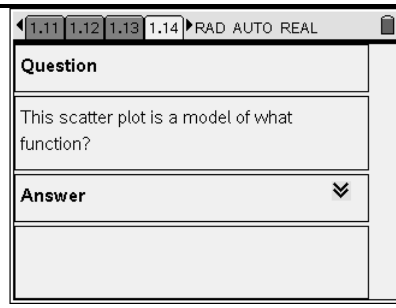
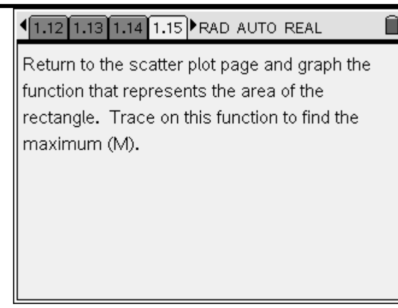
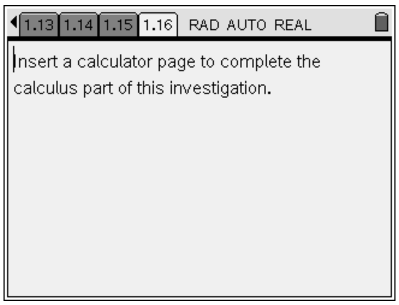

1.11 1.12 1.13 1.14 RAD AUTO REAL
Question
This scatter plot is a model of what function?
Answer


1.12 1.13 1.14 1.15 RAD AUTO REAL
Return to the scatter plot page and graph the function that represents the area of the rectangle. Trace on this function to find the maximum (M).


1.13 1.14 1.15 1.16 RAD AUTO REAL
Insert a calculator page to complete the calculus part of this investigation.