
 Here's Looking at Euclid – 8615

 Time required
 30 minutes

Topic: Numbers & Number Systems

- Determine the GCD and LCM of any pair of integers of two or fewer digits.
- Determine the GCD and LCM of any pair of integers of three or more digits.
- Express any positive integer as a product of prime powers.

Activity Overview

In this activity, you will explore:

- Creating a table of values to explore the remainder function.
 - Using Euclid's algorithm to find the GCD and a formula to find the LCM.
 - Using the GCD and LCM commands.
-

Teacher Preparation

This activity is designed for use in an Algebra 2 classroom, or as an extension for Algebra 1. It is best used to review factorization, GCD and LCM. The algebraic approach prepares students for similar operations over other fields, such as factoring monomials and polynomial division.

- *Prior to beginning the activity, students should know the definitions of greatest common divisor and least common multiple.*

Classroom Management

- *This activity is intended to have students explore **individually and in pairs**, (with the teacher providing assistance when needed). However, an alternate approach would be to introduce each method in a whole-class format, then have students apply the method individually or in pairs.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress.*
- *This worksheet serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*

TI-84 Plus Applications

none

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ID:8615

In this activity, you will explore:

- Creating a table of values to explore the remainder function.
- Using Euclid's algorithm to find the GCD and a formula to find the LCM.
- Using the GCD and LCM commands.

To start, clear out **Y1** and **Y2**. Use this document as a reference and to record your answers.

Euclid was a Greek mathematician who lived around 300 BC. In this activity, you will explore one of Euclid's many contributions to mathematics: an algorithm for finding the greatest common divisor of two numbers.

Enter the command **fPart(A/B)*B** in **Y1**. To access the **fPart** command, press **MATH**.

Arrow over to the **NUM** menu, and choose **fPart**(. Press **ALPHA** then **A** or **B** to type the variables.

```

Plot1 Plot2 Plot3
\Y1 =
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =
\Y7 =
    
```

```

Plot1 Plot2 Plot3
\Y1 fPart(A/B)*B
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =
    
```

```

MATH NUM CPX PRB
1: abs(
2: round(
3: iPart(
4: fPart(
5: int(
6: min(
7: max(
    
```

Now set a equal to 21.

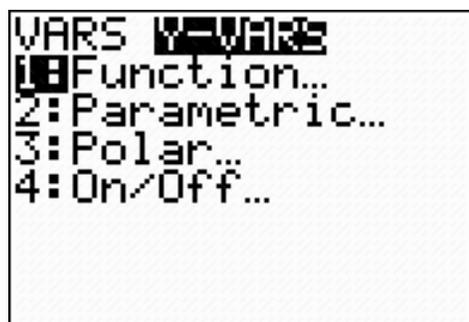
Press $21 \rightarrow \text{STO} \rightarrow \text{ALPHA} \rightarrow \text{A} \rightarrow \text{ENTER}$.



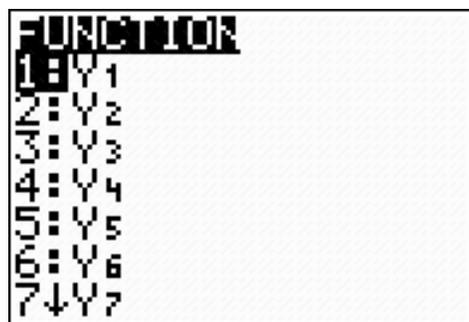
Set b equal to 5.



Find the value of the operation for these values of a and b . Press VAR . Arrow over to **Y-VARS**. Press ENTER to choose **Function**.



Press ENTER to choose **Y1**.



Press ENTER to evaluate **Y1** for these values of a and b .



Repeat the process with the values below and a few values of your own. Record your results in the table.

- The operation in **Y1** is called the **remainder** operation. It is also known as the *modulus* or *mod*. Look at the values in the table. What does the remainder operation do?
- Does the order of the two numbers matter?
- Is the remainder operation a function? Why or why not?

a	b	Y1
21	5	
21	7	
303	100	
303	3	

Once you understand the remainder operation, you are ready to use Euclid's algorithm. Use it to find the GCD of 385 and 280.

First, find the remainder when the larger number is divided by the smaller number. Use the remainder operation with $a = 385$ and $b = 280$.

385	→A	385
280	→B	280
Y1		105

Now b becomes the new a , and the remainder becomes the new b . Find the remainder when 280 is divided by 105.

280	→A	105
105	→B	280
Y1		105
		70

Repeat the process. Again the old b becomes the new a and the remainder becomes the new b .

Keep going until the remainder is 0. The last remainder before 0 is the GCD. In this case, $\text{gcd}(385, 280) = 35$.

- Could the algorithm continue once a remainder of 0 is reached? Why or why not?

105	→A	70
70	→B	105
Y1		70
		35

Use Euclid's algorithm to find the GCD of each pair of numbers. Record your answers in the table. You will complete the LCM column later.

- Are any of the pairs of numbers in the table relatively prime? How do you know?
- Describe what happens when you use Euclid's algorithm when a is a multiple of b .

a	b	GCD	LCM
21	5		
21	7		
303	100		
303	3		

Once you know the GCD, you can use the formula below to find the LCM.

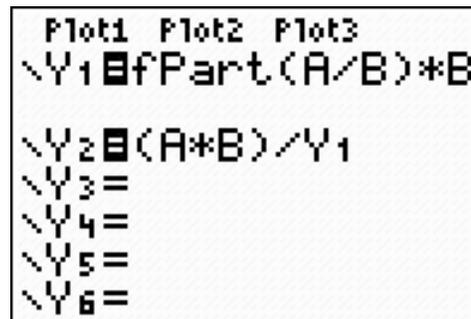
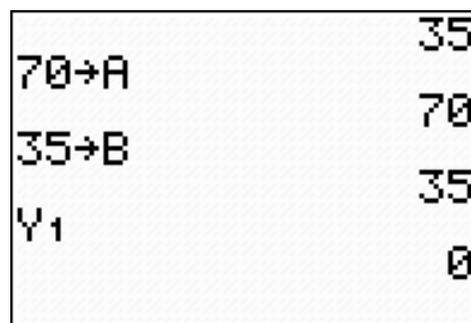
$$\text{lcm}(a,b) = \frac{a \cdot b}{\text{gcd}(a,b)}$$

Enter this formula into your calculator as **Y2**. Use parentheses around the numerator, and use **Y1** for the denominator.

Use **Y2** to complete the LCM column in the table above.

- What is the LCM of two numbers that are relatively prime?
- What is $\text{lcm}(a, b)$ when a is a multiple of b ?

Your calculator has commands to calculate the GCD and LCM. They are found in the **Math > Num** menu.



Use these commands to check the answers in the table.

- Does the order of the numbers in the GCD or LCM command matter?

```
gcd(385,280)      35
lcm(385,280)     3080
```