EXPLORATIONS

Activity 8

Power Patterns

Concepts/Skills:

Roots and powers, problem solving

Calculator:

TI-30Xa SE or TI-34

Objectives:

Students will investigate patterns that show relationships between powers and roots.

Getting Students Involved

Finding patterns in data is a very important part of everyday life.

Discuss any activity you may have completed earlier in the year in which students gathered data and made sense of the information.

Help students recall what strategies they used to find the important patterns in those data.

Making Mathematical Connections

As a number increases, the roots (for example, square root, cube root) also increase. Also, if X is between A and B, then the root of X is between the root of A and the root of B.

- ♦ We know that the square root of 49 is 7 and the square root of 64 is 8. What can you say about the square root of 55? *It is between 7 and 8.*

Review the size of the display on the calculator.

- ♦ How many digits are there in the display on your calculator? What is the greatest number that you can display on the calculator without using scientific notation? *Answer will vary, depending on the calculator being used.*

Be sure that students know how to use the following keys:

$\boxed{x^2}$ $\boxed{\sqrt{x}}$ $[x^3]$ $[\sqrt[3]{x}]$ $\boxed{y^x}$ $[\sqrt[x]{y}]$

➠ **Transparency Masters** F: Square and Square Root
G: Cube and Cube Root
H: Powers and Roots

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Carrying Out the Investigation

Some students may not know immediately how to use information in the tables to help them answer questions **2**, **4**, **8**, and **10**. Remind them that if $a<x<b$, then $a^2<x^2<b^2$ and $\sqrt{a}<\sqrt{x}<\sqrt{b}$.

- You know that $\sqrt{900}$ is 30 and $\sqrt{1,000}$ is a little more than 31.6. Is there any whole number greater than 30 whose square would be less than 1,000?

  $31^2 = 961$

Students may want to solve the riddles through trial and error. Help them see how to use logical reasoning as a more efficient strategy.

Making Sense of What Happened

Ask student to explain how they found their answers to the questions.

Probe how students transfer the strategies they used on the first two pages to help them solve the riddles on the last two pages.

- Which information from the tables did you use to answer the questions?

  Possible answers: Look for whole numbers in between the roots in the table, identify the least whole number larger than the largest root in the table, ignore the decimal parts of the roots.

- Did you treat the whole number roots differently than the roots that contained decimal parts?

  Possible answer: I tried to think of other numbers which would have whole number roots.

- How did you use the largest root in each table?

  The next larger whole number would have a power greater than any number in the table.

- How did answering the questions on the first two pages help you solve the riddles?

  Think of ways to make similar tables related to the clues.

Continuing the Investigation

Encourage students to create additional tables or additional riddles like those in the activity.

- What numbers would you put in a table for 10th roots? For 15th roots?

  Possible answer: Begin with 1,000 since $2^{10} = 1024$ (or 32,000 since $2^{15} = 32,768$).

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Solutions

Only four decimal places are provided for roots. The number of decimal places displayed by the calculator depends on the particular calculator being used.

1.

Number	Square Root
100	10
200	14.1421...
300	17.3205...
400	20
500	22.3606...
600	24.4948...
700	26.4575...
800	28.2842...
900	30
1,000	31.6227...

2. $31^2 = 961$. Since $30 < 31 < 31.662$, then $30^2 < 31^2 < 31.6227^2$. That is, 31^2 is between 900 and 1,000. The next greater perfect square ($32^2 = 1{,}024$) is greater than 1,000.

3.

Number	Cube Root
1,000	10
2,000	12.5992...
3,000	14.4224...
4,000	15.8740...
5,000	17.0997...
6,000	18.1712...
7,000	19.1293...
8,000	20
9,000	20.8008...
10,000	21.5443...

4. $21^3 = 9{,}261$. Since $20.80 < 21 < 21.54$, 9,261 is the greatest perfect cube less than 10,000.

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5. $99999^2 = 9999800001$, since 100000^2 is displayed as 1 ^{10}.

One way to find the answer is to put the greatest number in the display, take the square root, and experiment with the number shown and one less than the number shown. Remember that the calculator may "round" the root, so it may display a value that is slightly greater than the actual value.

6. $2154^3 = 9993948264$, since 2155^3 is displayed as 1.000787388 ^{10}.

7.

Number	Fourth Root
10,000	10
20,000	11.8920...
30,000	13.1607...
40,000	14.1421...
50,000	14.9534...
60,000	15.6508...
70,000	16.2657...
80,000	16.8179...
90,000	17.3205...
100,000	17.7827...

8. $17^4 = 83521$, since 18^4 is greater than 100,000.

9.

Number	Seventh Root
10,000,000	10
20,000,000	11.0408...
30,000,000	11.6993...
40,000,000	12.1901...
50,000,000	12.5849...
60,000,000	12.9170...
70,000,000	13.2046...
80,000,000	13.4590...
90,000,000	13.6873...
100,000,000	13.8949...

10. $13^7 = 62{,}748{,}517$, since 14^7 is greater than 100,000,000.

11. $316^4 = 9971220736$, since 317^4 is displayed as 1.009803912 ^{10}.

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12. 26^7 = 8,031,810,176, since 27^7 is displayed as 1.04603532 ^{10}.

13. 17

14. 14

15. 28

16. 19 (20^3 = 8,000, but we need a cube less than 8,000.)

17. 15

18. 1,024

19. 10,648

20. 1 + 2 + 3 + 4+ 5 + 6 + 7 + 8 + 9 + 10 = 55

21. 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 = 385

22. 1 + 8 + 27 + 64 + 125 + 216 + 343 + 512 + 729 + 1000 = 3,025

23. 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29 = 129

24. 4 + 9 + 25 + 49 + 121 + 169 + 289 + 361 + 529 + 841 = 2,397

25. 8 + 27 + 125 + 343 + 1331 + 2197 + 4913 + 6859 + 12167 + 24389 = 52359

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