Altitude to the Hypotenuse

Math Objectives

- Students will investigate and state the relationship between the altitude to the hypotenuse and the two segments of the hypotenuse formed by this altitude.
- Students will write a conditional statement representing this relationship.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- altitude (height of triangle)
- geometric mean
- hypotenuse

About the Lesson

- This lesson involves observing changes in a construction of a right triangle. Students will progress from manipulating objects, describing observations, and inferring relationships, to making deductive arguments to state a theorem. As a result, students will:
    - Discover the relationship between the altitude to the hypotenuse and the two segments on the hypotenuse cut by this altitude.
    - Discover that the length of the altitude is the geometric mean between the lengths of the two segments forming the hypotenuse.

Prerequisite Knowledge

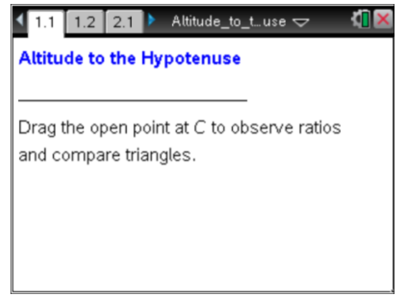
- Pythagorean Theorem
- Writing proportions for similar triangles and conditional statements

TI-Nspire™ Navigator™

- Live Presenter
- Quick Poll

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Altitude_to_the_Hypotenuse_Student.pdf
- Altitude_to_the_Hypotenuse_Student.doc

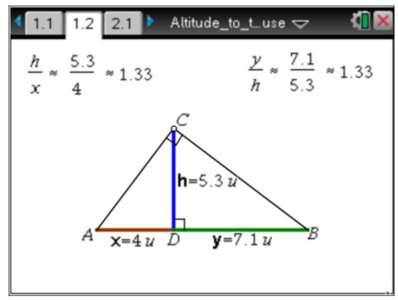
TI-Nspire document

- Altitude_to_the_Hypotenuse.tns

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand ( ) getting ready to grab the point. Press ctrl to grab the point and close the hand ( ).

Move to page 1.2.



1. Examine the angle markings of the sketch.

   a. What kind of triangles are Δ*ACB*, Δ*ADC,* and Δ*BDC*? Explain how you know.

   Answer: They are right triangles. ∠*ACB* and ∠*BDC* are marked as right angles. ∠*ADC* and ∠*BDC* form a linear pair, so they are supplementary, which means ∠*ADC* is also a right triangle.

   b. Name all of the altitudes of Δ*ACB* that are shown in this sketch. Justify your answers.

   Answer: $\overline{CD}$ is an altitude; one endpoint is vertex *C;* and it is perpendicular to $\overline{AB}$, which is the side opposite vertex *C*.
   $\overline{AC}$ is an altitude; one endpoint is vertex *A;* and it is perpendicular to $\overline{BC}$, which is the side opposite vertex *A*.
   $\overline{BC}$ is an altitude; one endpoint is vertex *B;* and it is perpendicular to $\overline{AC}$, which is the side opposite vertex *B*.

   c. Which one of the altitudes of Δ*ACB* shown is the altitude to the hypotenuse?

   Answer: $\overline{CD}$

2. Drag the open circle at point *C*.

   a. What stays the same as you drag point *C*?

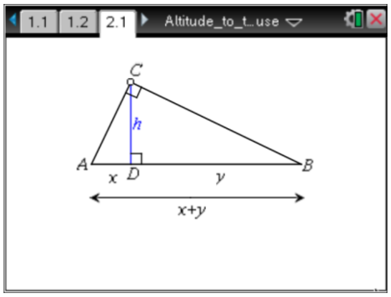
   Answer: The three triangles all remain right triangles and the measure of $\overline{AB}$ remains the same. Also, the ratios $\frac{x}{h}$ and $\frac{h}{y}$ are always equal.

Teacher Tip: At this point you have not proved that the ratios are equal; you have observed that these ratios were equal in all the examples you examined. This is an excellent time to discuss inductive versus deductive reasoning. Now that you have induced that these ratios are always equal, you can use deductive reasoning to make a convincing argument.

b. What changes as you drag point *C?*

Answer: The measures *x*, *y,* and *h* all change and the ratios $\frac{x}{h}$ and $\frac{h}{y}$ change.

Move to page 2.1.



3. Examine the sketch.

What variable represents the measure of each of the following?

Shorter leg of ΔADC: *x*
Longer leg of ΔADC: *h*
Shorter leg of ΔBDC: *h*
Longer leg of ΔBDC: *y*

Teacher Tip: This question is designed to get students thinking about corresponding sides before they get to question 5.

4. Drag the open circle at point *C*. What happens?

Answer: ΔADC rotates around point *D*.

5. Drag the open circle at point *C* until $\overline{AD}$ is on top of $\overline{CD}$ and $\overline{CD}$ is on top of $\overline{BD}$.

a. Write a similarity statement for the two smaller right triangles and explain why these triangles are similar.

Answer: Answers may vary; for example, $\Delta ADC \sim \Delta CDB$. Reasons will vary depending on which similarity theorem students use. If AA similarity is used, they may make the argument that $\angle ADC$ and $\angle CDB$ are congruent since they are both right angles, $\angle CAD$ is complementary to $\angle ACD$ and $\angle BCD$ is complementary to $\angle ACD$, so $\angle CAD$ is congruent to $\angle BCD$. There are many different but correct arguments that can be made.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

Teacher Tip: Students may try to use SAS similarity since they saw on page 1.2 that the ratios were always equal. This relationship was observed but not justified, so it cannot be used here to justify the fact that the two small right triangles are similar.

b. How does the fact that the two small triangles are similar justify the fact that ratios $\frac{x}{h}$ and $\frac{h}{y}$ are always equal?

Answer: The corresponding sides of similar triangles are proportional.

6. Use algebra to solve the equation $\frac{x}{h} = \frac{h}{y}$ for *h.*

Answer: $\boldsymbol{h = \sqrt{xy}}$

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Teacher Tip: Since *h, x,* and *y* are all distance measures, the negative square root does not have to be considered.

7. Drag the open circle at the original point C until the thick copy of $\overline{AD}$ is equal to $\overline{CD}$. What is the relationship between *x, y*, and *h* now?

Answer: They are all equal.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 3 at the end of this lesson.

Teacher Tip: Notice that the length of the altitude to the hypotenuse of a right triangle (*h*) is the geometric mean of the lengths of the segments (*x* and *y*), whose sum is equal to the length of the hypotenuse. That is, if students are given two numbers, *x* and *y*, whose product is always h^2, and $x = y$, then *x* and *y* must be equal to *h.*

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The relationship between the measure of the altitude to the hypotenuse and the measures of the two segments formed by this altitude intersecting the hypotenuse.
- How to write a conditional statement representing this relationship.

Assessment

Using the diagram on page 2.1 with question 3, suppose that $x = 4$ and $y = 5$. What is the value of h?

Answer: $\frac{4}{h} = \frac{h}{5} \Leftrightarrow h = 2\sqrt{5}$

Give students other exercises where two of the measures (*h, x, y*) are given and they need to find the missing measure.

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Note 1

Question 5a, *Live Presenter*: You might make a student the Live Presenter to have the student demonstrate how to move ΔADC on top of ΔCDB. Then students can write the similarity statement.

Note 2

Question 6, *Quick Poll*: Send an Open Response Quick Poll to collect students' responses to this question.

Note 3

Question 7, *Live Presenter*: Make a student the Live Presenter and have the student demonstrate how to move the original point *C* so that the thick copy of $\overline{AD}$ is congruent to $\overline{CD}$. Then students can make their conjectures. The Live Presenter can also demonstrate how to measure the segments to verify students' conjectures.