NUMB3RS Activity: Mapping Fireprints
Episode: "Scorched"

Topic: 3-dimensional graphing **Grade Level**: 9 – 12
Objective: Students will find the equation of a plane, using three variables.
Time: 20 minutes

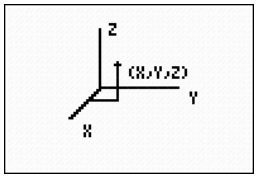
Introduction

When determining relationships among objects or events, it is necessary to look at the variables that describe them. For example, when comparing two fires, we might measure burn rate, length of scorch marks, and heat intensity at the source. These variables may be related, and the relationship can help to identify fires that have a common cause.

Discuss with Students

You should be familiar with points being described by two variables, x and y: for example, length of a person's foot (x) compared to his or her height (y). If you wanted to see whether there was a relationship between these two variables, you might look at several people (points), find the length of their feet and corresponding heights, and plot it on a graph. If we can find a line that comes close to containing all of the points, then the equation of that line shows how the variables *foot length* and *height* are related for this sample population.

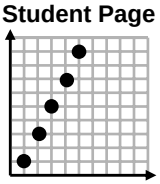
Sometimes, we have more than two variables to consider. In this activity, points are described by three or more variables. For example, to describe a fire you might investigate burn rate, length of scorch marks, and heat intensity. For three variables, we can use x, y, and z as the coordinates. Just like points with two variables can be graphed in two dimensions, points with three variables can be graphed in three dimensions. To visualize points in three dimensions, think about bubbles in the air that do not move. From a front corner of the room, a bubble is located a distance out from the corner along a side wall, over a distance parallel to the front wall, and a height up from the floor. The location of this point is written as an ordered triple (x, y, z).



If several bubbles form a linear relationship in three dimensions, you can use a plane to describe the points. Points that lie on the same plane share a linear relationship, just like points on a line. If we can find the equation of the plane, then we can learn how the different variables relate to each other.

In this activity, you will need to review how to solve a system of linear equations. The "Extensions" page will describe a more elegant way of finding the equation of a plane, using vectors.

Student Page Answers:



1. $y = 2x - 1$ **2.** *yes* **3.** $z = 7;\ y = 1;\ x = 2$ **4a.** $c = 3$ **4b.** $a = 2,\ b = -1$ **4c.** $z = 2x - y + 3$ **5.** $5 = 2 - 0 + 3;\ 4 = 2(2) - 3 + 3;\ 4 = 2(1) - 1 + 3;\ 0 = 2(-1) - 1 + 3;\ 3 = 0 + 0 + 3;\ 2 = -1 + 3$ **6.** $5 = 4(2) + 2(-1) - 1,\ 5 = 2 - (-1) + 2;\ y = -2,\ z = 7;$ *a line* ***7.*** *Each linear equation removes one variable. 600 equations are needed.*

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Name: _____ Date: _____

NUMB3RS Activity: Mapping Fireprints

Charlie wants to find out if a string of fires was set by the same person. Larry tells Charlie, "If every arsonist approaches a fire differently, it makes sense that every fire set would be unique to the person setting it. Not just in terms of the method but in terms of the entire character of the fire. What we're talking about is a fingerprint for fires." Charlie calls this unique pattern a *fireprint*. If Charlie is able to find the arsonist's fireprint, he can then find other fires set by the same person. Let's explore how this is done. We will start with a simple example.

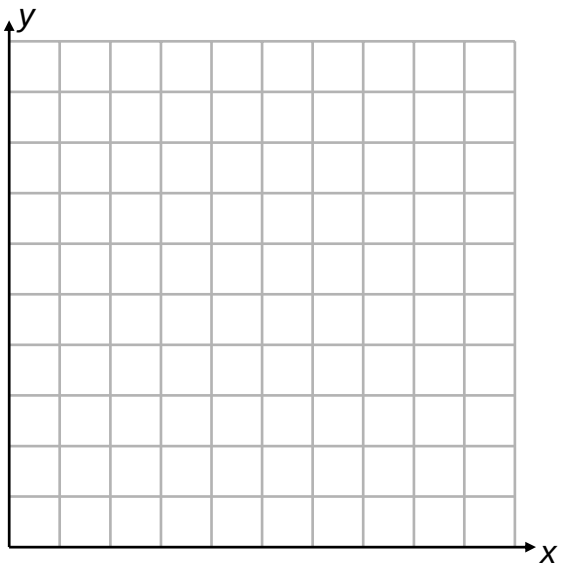
Charlie has collected two pieces of data for each of five fires set by the same person: rate of burn (feet per minute) and length of scorch marks (feet). He lists the data as a set of values:

(rate of burn, length of scorch marks)

The five fires have the following values for rate of burn and length of scorch marks:

(2, 3), (4, 7), (1, 1), (5, 9), (3, 5).

Plot these points on the graph at right.



You should notice that the data points form a line. The equation of the line is the fireprint of the arsonist setting the fires.

1. Determine an equation of the line you drew above: _____

2. Using the equation you found in #1, determine if a fire whose data is (10, 19) fits the fireprint. _____

As you can imagine, real arson investigations are much more complicated than this. If we add one more variable to study, the result of a fireprint will no longer be a line. With three variables, we look for a *plane* that contains all of the points.

If Charlie has three data values for each fire (x = burn rate, y = length of scorch marks, and z = heat intensity at the source), each point (x, y, z) would be plotted on a 3-dimensional graph. If these points are to form a "fireprint," then they will need to lie in a pattern – this time on a plane. The equation of this plane (the fireprint) is a linear equation, like the equation of straight line, but has three variables: x, y, and z.

3. In one fireprint, the three variables are related by the equation $\boldsymbol{z = x + 3y - 2}$.
If $x = 3$ and $y = 2$, what is z? _____
If $x = 2$ and $z = 3$, what is y? _____
If $y = 0$ and $z = 0$, what is x? _____

4. Charlie has the following data from six recent fires:

$\boldsymbol{A = (1, 0, 5),\ B = (2, 3, 4),\ C = (1, 1, 4),\ D = (3, 1, 8),\ E = (0, 0, 3),\ F = (0, 1, 2)}$

He wants to find the constants a, b, and c so that each point satisfies the same equation: $\boldsymbol{z = ax + by + c}$.

a) To find the value of c, substitute the coordinates of point E for x, y, and z, and solve for c. _____

b) Find a and b by substituting the coordinates of any two points other than E and solving the system of equations. _____

c) Using your values for a, b and c, write the equation of this fireprint.

_____

5. Show that all six of the points satisfy the fireprint equation you found in #4.

_____

_____

6. Consider the points that satisfy two different linear equations in three variables:

$$\boldsymbol{z = 4x + 2y - 1} \text{ and } \boldsymbol{z = x - y + 2.}$$

Show that (2, –1, 5) is a point that satisfies both of these equations: _____

_____

Find values of y and z so that the point (3, y, z) lies on both planes listed above. _____

These two equations, taken together, give us a more restrictive fireprint. What geometric object corresponds to the set of points that lie on both of these planes? _____

7. Challenge: Charlie starts with 600 variables. How many linear equations will he need to find the constants for the equation of this fireprint? (Hint: How many equations were needed to find the constants for two variables? How many equations were needed for three variables?) _____

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Find the Equation of a Plane Using Vector Cross Products

Suppose you want to find the fireprint (equation of the plane) using the data points:

***A* = (1, 0, 5), *B* = (2, 3, 4), *C* = (1, 1, 4), *D* = (3, 1, 8), *E* = (0, 0, 3), and *F* = (0, 1, 2)**

One way to find the fireprint would be to solve systems of equations using the same method shown in the activity. A more elegant way to find the equation of a plane is through the use of vectors and cross products. To learn how to use vectors to find the equation of a plane, use the links below to explore topics such as vectors, cross products, and determinants.

Vectors: **http://mathforum.org/~klotz/Vectors/vectors.html**
Equation of a plane:**http://tutorial.math.lamar.edu/AllBrowsers/2415/EqnsOfPlanes.asp**

Principal Components Analysis

In this episode, Charlie uses Principal Components Analysis (PCA) to study his data. For real data in 3-dimensional space, the points might lie very close to a single plane without all being on the same plane. PCA is a mathematical technique based on methods of linear algebra that finds the plane that best fits all of the data points. It can work with any number of variables, even the 600 variables that Charlie has identified. There is more information on PCA at **http://ordination.okstate.edu/PCA.htm**

Related Topic: 3-dimensional graphing

Go to the website **http://cs.jsu.edu/mcis/faculty/leathrum/Mathlets/grapher3d.html** and input the fireprint from the extensions solved for *z*. Then click on the graph and drag the axis in different directions. This is the fireprint-plane. Try other equations (with exponents or trigonometric functions) and see what their graphs look like.

You can perform 3-dimensional graphing in the palm of your hand with a TI-89 or Voyage 200 Graphing Calculator. The Texas Instruments website has help and screen shots of how to do this.
http://education.ti.com/us/product/tech/89/down/89tips-08.html

To further research 3-dimensional graphs and how they are used for analysis, visit the website below.
http://www.sv.vt.edu/classes/surp/surp96/laughlin/stat/3D_tutor/3D_tutor.html

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