Secant and Tangent Lines
ID: 11140

Time Required
15 minutes

Activity Overview
In this activity, students will observe the slopes of the secant line and tangent line as point Q on the function approaches the other point P. They will also determine the average rate of change for an interval and approximate the instantaneous rate of change using the slope of the secant line.

Topic: The Concept of the Derivative
- *Average Rate of Change of a Function*

- *Instantaneous Rate of Change of a Function*

Teacher Preparation and Notes
- *The accompanying student worksheet supplies key press, step-by-step instructions, and questions.*

- *Students are introduced to interpreting the slope of the secant line as the average rate of change of a function over a given interval and that the slope of the secant line approaches the slope of the tangent line as point Q approaches point P. Students are also introduced to the concept of the slope of the tangent line representing the instantaneous rate of change of a function at a given value of x. The instantaneous rate of change of a function can be estimated by the slope of the secant line. This estimation gets better the closer point Q gets to point P. (Note: This is only true if y1(x) is differentiable at point P.)*

- ***To download the student worksheet, go to education.ti.com/exchange and enter "11140" in the quick search box.***

Associated Materials
- *CalcWeek03_SecTan_worksheet_TI89.doc*

Suggested Related Activities

- *The Tangent Line Problem — 8315*
- *The Mean Value Theorem — 9896*

Problem 1 – Graphing secant and tangent lines

Students are introduced/reminded of what a secant line is. Even though points P and Q are NOT labeled on their graph, they will explore the slope and how to make point Q approach P.

Students are to first graph $y1(x) = x^2$, adjusting the window settings to those given on the worksheet.
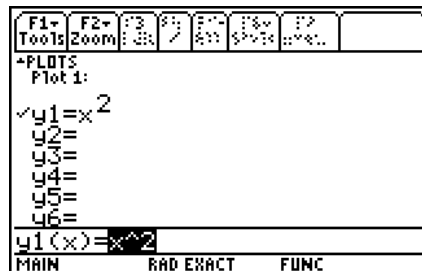
Using the graph screen's **Math** menu ([F5]), students will have a tangent line drawn at point $P(1,1)$. The equation of the line will appear on the screen.
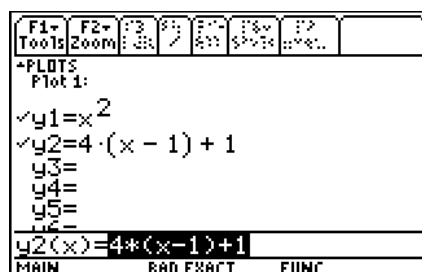
Note: $y = 2(x - 1) + 1 = 2x - 2 + 1 = 2x - 1$
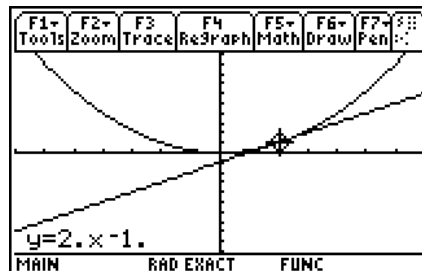
Now students will graph the secant line that passes through (1, 1) and (3, 9). They should show how they got the equation for the graph before entering it in $y2(x)$.

$$y - y_1 = m^*(x - x_1)$$
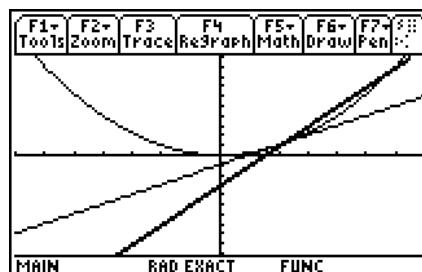
$$y - 1 = 4(x - 1)$$

$$y = 4(x - 1) + 1$$

If they are having trouble seeing the points of intersection, have them adjust the window setting to zoom in on the first quadrant.

Note: The dark line is the secant line. To change the attributes of a line, on the Y= screen press [2nd] [F1] and select **Thick**.

Discuss with students how the secant and tangent lines compare to each other. (secant line is steeper)

Students are to change the slope of the secant line to 3.5, 3, and 2.5, viewing the graph after each. They will need to re-graph the tangent line.

They should discover that the slope of the secant line approaches the slope of the tangent line as point Q approaches point P.
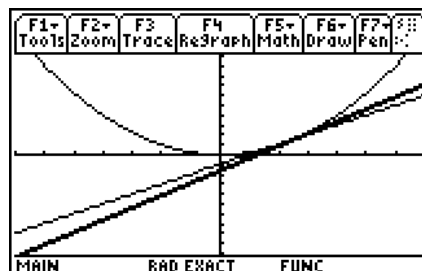
Problem 2 – Comparing Slopes Numerically

Students are asked to view the table of values of $y1(x)$ and use the calculator (Home screen) to find the average rate of change of $y1(x)$ on the interval [1, 1.1].

They will most likely enter the values directly as shown in the first calculation in screenshot to the right. The second calculation is using function notation. Either way, students should arrive at the same answer.

Problem 3 – Comparing Slopes Algebraically

Students are asked to write an expression to estimate the instantaneous rate of change of $y1(x)$ at $x = 1$ and then use the calculator HOME screen to evaluate the expression.

They can get a better estimate by using the slope of the secant line containing point P and another point closer to P than

The example shown uses the point (1.001, $y1$(1.001)).

Extension – Rectilinear Motion Application

Before beginning, students can define the functions to make the calculations easier. The **Define** command is found in the **Other** menu (F4).

Introduce the terms *average velocity* and *instantaneous velocity* for the first two questions. The answers will have units **meters per second** (*m*/*s*).

Introduce the terms *average acceleration* and *instantaneous acceleration* for the last two questions. The answers will have units **meters per second squared** (*m*/*s*^{2}).