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Textbook Correlation: Key Topic

- Normal Distribution
- Univariate Data: Measures of Center (Mean)
- Univariate Data: Measures of Spread (Standard Deviation)
- Univariate Data: Measures of Position (Percentiles)

NCTM Principles and Standards:

- Process Standard
- Representation
- Connections
- Problem Solving


## Description

The purpose of this activity is to apply the normal density function, the normal cumulative distribution function, and the inverse normal cumulative distribution function to analyze a population with a normal distribution.

The distribution of heights of a population is approximately normal with mean $\mu=65.5$ and standard deviation $\sigma=2.5$ inches. From the 68-95-99.7 rule we can estimate that the middle 99.7\% (almost all) of the distribution lies between 58 and 73 inches.

## a) What percent of the population are between 63 and 68 inches tall?

## Solution:

The normal curve for this distribution can be graphed on the TI-89 as illustrated in the screens below. After setting $\mathbf{x m i n}=58, \mathbf{x m a x}=73$, and $\mathbf{x s c l}=2.5$, the ZoomFit option sets the $y$ values for the best fit on the screen.


The Shade Normal (shadnorm) command draws the normal density function specified by mean and standard deviation, and shades the area under the normal curve between the lower value and upper value as depicted with two methods in the following screens.

Method I - on the HOME Screen.


Method II - in the List Editor.


Answer: Approximately $\mathbf{. 6 8 2 6 8 9}$ or $\mathbf{6 8 \%}$ of the population are between 63 and 68 inches tall. Note that these values are within 2 standard deviations of the mean.

## b) What percent of the population are less than 69 inches tall?



Answer: Approximately $\mathbf{. 9 1 9 2 4 3}$ or $\mathbf{9 2} \%$ of the population are less than 69 inches tall.
c) What percent of the population are over 70.5 inches?



Note that the normal cumulative distribution function, "normcdf" command can be used to obtain the same result without drawing a graph as illustrated above on the Home Screen. It can also be executed in the List Editor as shown in the following screens.


Answer: Approximately $\mathbf{. 0 2 2 7 5}$ or $\mathbf{2 \%}$ of the population are over 70.5 inches.

## d) What are the heights of the tallest $5 \%$ of the population?

Solution: The Inverse Normal command computes the inverse cumulative normal distribution function for a given area under the normal distribution curve specified by mean and standard deviation. Since the area under the normal curve is computed from the left hand side and the total area under the normal curve is 1 , we need to look for what height separates the tallest $5 \%$ from the other $95 \%$ or, eqivalently, the $95^{\text {th }}$ percentile. On the HOME Screen entry line paste invNorm and enter $\mathbf{. 9 5}$ or ( $1-.05$ ) for area, $\mathbf{6 5 . 5}$ for mean $\mu$ and $\mathbf{2 . 5}$ for standard deviation $\sigma$. The following screens depict two methods for executing the invNorm Command.

Method I - on the Home Screen


Method II - in the List Editor


Method II - in the List Editor


Answer: The tallest 5\% of the population are 69.61 inches or taller.

The following screen illustrates how to confirm the answer with the normCdf command.


## References

Connors, M.A. and Connors, E.A. (in press). "Statistics With The TI-89 Statistics Flash Application And List Editor," Proceedings of the Fourteenth International Conference on Technology in Collegiate Mathematics, Baltimore, MD, November 1-4, 2001. Reading, MA: Addison Wesley.

