Piecewise Functions, Continuity, and Differentiability

by

Mary Ann Connors

Department of Mathematics
Westfield State College
Westfield, MA 01086

Textbook Correlation: Key Topic

- Pre-Requisites: Functions and Equations
- Derivatives
- Limits and Continuity

NCTM Principles and Standards:

- Process Standard
  - Representation
  - Connections

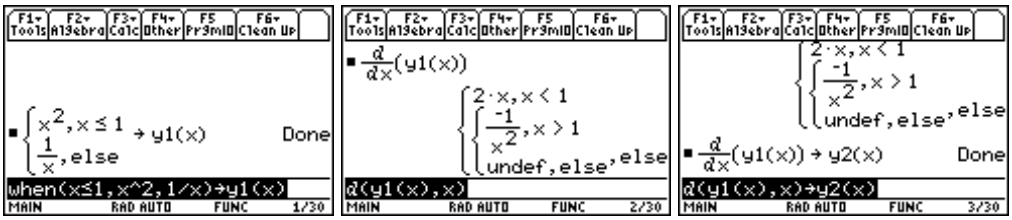
Exercises:

1. Consider the function given by $f(x) = \begin{cases} x^2, x \le 1, \\ 1/x, x > 1 \end{cases}$.

   Graph f and its derivative, f'.

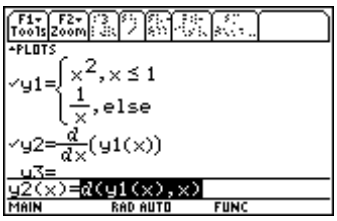
   Solution:

   Reproduce the following screens on your TI-89.

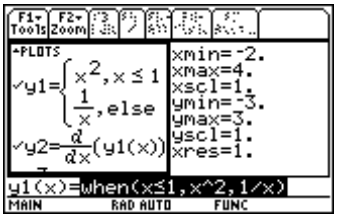


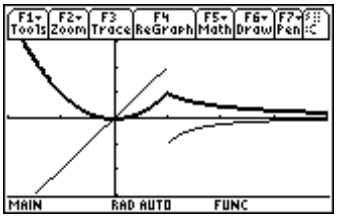
   On the home screen use the **when** and **STO>**commands for the first condition of the piecewise function and store it in $y1(x)$. Find the derivative of $y1$ and store it in $y2(x)$.
   Note: Nested **when** commands define the three-part rule for the derivative.
   This derivative could also be entered as:
   Define y2(x) = when(x<1,2x,when((x>1,-1/x^2,undef)).

Another method to define $y1$ and $y2$ is portrayed below in the Y= editor.



Since different graphing styles are allowed for each function, use the dot style for the graph of the derivative function with jump discontinuity and thick for the graph of $y1$.



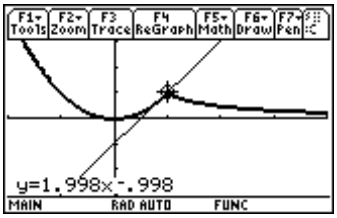


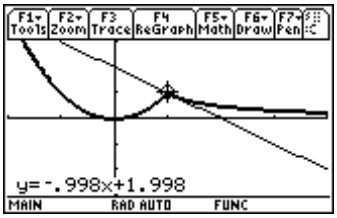
2. Is $y1$ continuous at $x = 1$?

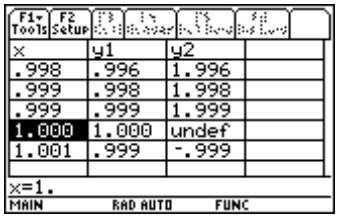
   Answer: Since $y1(1) = 1$ and $lim_{x \to 1} y1(x) = 1$, $y1$ is continuous at $x = 1$.

3. Is $y1$ differentiable at $x = 1$?

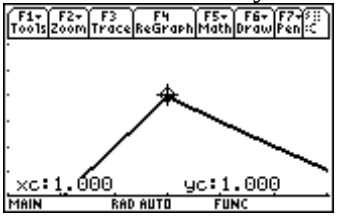
   Solution:
   Use the graph of the original function to explore the question of differentiability at $x = 1$. Draw the tangent lines at $x = .999$ and $x = 1.001$ to visualize the approximate values of the left and right hand derivatives at $x = 1$. Access the command by selecting **F5 (Math)**, **A: Tangent**. Type in the x value where the tangent line is to be drawn. Notice its equation in the bottom left hand corner of the screen. Look at the **TABLE**.







   Alternatively, zoom in three times at the point (1, 1) and notice that a corner appears. This illustrates that the function is not locally linear, or differentiable, at $x = 1$.



Additional Exercise:

Consider the function given by $f(x) = |x|$. Graph f and its derivative, f'.
Is f continuous at $x = 0$? Is f differentiable at $x = 0$?