EXPLORATIONS™

Activity 2

Objectives

- Given a function, state and explain the limit at a particular value
- Given a graph, state and explain the limit at a particular value

Materials

- TI-84 Plus / TI-83 Plus

Is There a Limit to Which Side You Can Take?

Introduction

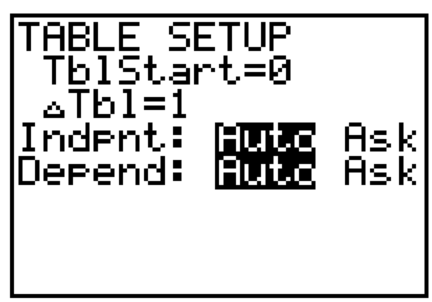
The limit describes the behavior of a function near a point. It represents how function outputs behave as inputs get very close to a value of interest. In some cases, the value of a limit depends on from which side the input value is approached. In this activity, you will investigate the idea of one-sided limits both graphically and numerically.

Exploration

1. Enter this piecewise function into your graphing handheld:

$$f(x) = \begin{cases} x - 3, x < 2 \\ x + 1, x > 2 \end{cases}$$

2. Set up your table as shown.



3. Using your knowledge of piecewise functions and the table output, record the function values for $x = \{1, 2, 3\}$.

4. Take a closer look at what happens as the input gets closer to 2. In other words, look at $\lim_{x \to 2} f(x)$.

Change the table to start at 1.7 and increment the table by 0.1. In the table at the right, record the values of $f(x)$ for the following inputs:

$x = \{1.7, 1.8, 1.9, 2, 2.1, 2.2, 2.3\}$

X	Y1	Y2
1.7		
1.8		
1.9		
2		
2.1		
2.2		
2.3		

X=2

5. Change the table to start at 1.97, and increment by 0.01. Record the values of $f(x)$ in the table shown.

X	Y1	Y2
1.97		
1.98		
1.99		
2		
2.01		
2.02		
2.03		

X=2

6. From your knowledge of limits, what is $\lim_{x \to 2} f(x)$? Explain.

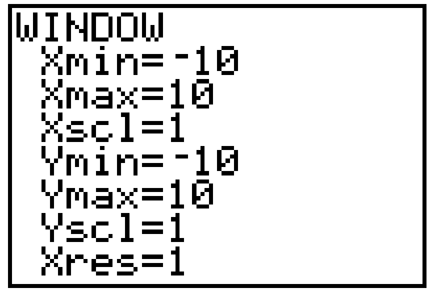
7. The notation $\lim_{x \to 2^+} f(x)$ means to investigate the limit of the function $f(x)$ as

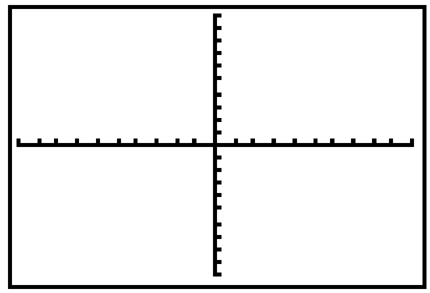
x approaches 2 through values that are greater than 2 (from the right). In this case, you would be looking at what happens as the input value gets *very near* 2 from values higher than 2. Using input values $x = \{2.3, 2.2, 2.1, 2\}$, what conclusion would you draw regarding $\lim_{x \to 2^+} f(x)$, and why?

8. Explain what the notation $\lim_{x \to 2^-} f(x)$ means.

Give some examples of useful input and a value for this limit, if any.

9. Confirm what you concluded by sketching the graph on the axes at the right, using the standard viewing window.





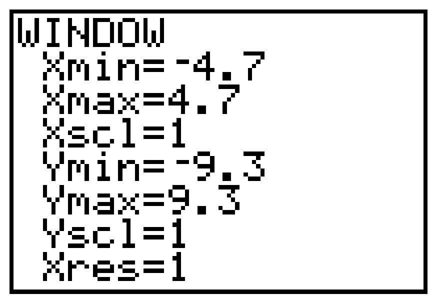
Imagine examining $\lim_{x \to 2^+} f(x)$ by "walking" from the right along the proper branch of the graph toward the value $x = 2$, and examining $\lim_{x \to 2^-} f(x)$ by walking from the left along the proper branch toward the input value $x = 2$.

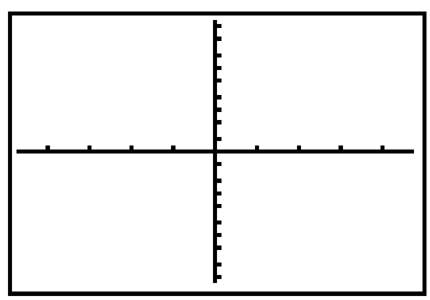
10. Use any zooming technique you prefer, keeping both branches visible and keeping $x = 2$ toward the center of the window. Trace along each branch. What do you see as the result?

11. Graph the function

$$g(x) = \begin{cases} x+1, x > 2 \\ 5, x = 2 \\ x-3, x < 2 \end{cases}$$

with the **WINDOW** settings shown. Sketch what you see.





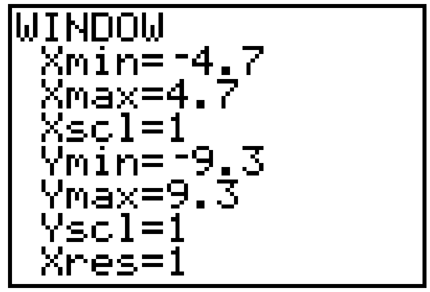
12. What difference, if any, is there in $g(x)$ from $f(x)$?

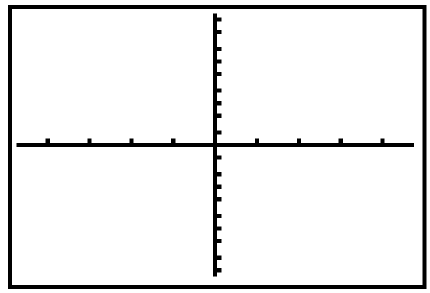
13. Find the following limits, and explain your results: $\lim_{x \to 2} g(x)$, $\lim_{x \to 2^+} g(x)$, $\lim_{x \to 2^-} g(x)$.

14. Graph the function

$$h(x) = \begin{cases} \dfrac{1}{(x+2)}, & x < -1 \\ x^2 + 2, & -1 \le x < 3 \\ -x + 9, & x \ge 3 \end{cases}$$

in the viewing window given, and sketch what you see.





15. Find each limit, and explain how you arrived at your conclusion.

a. $\lim_{x \to -2} h(x)$

b. $\lim_{x \to -2^+} h(x)$

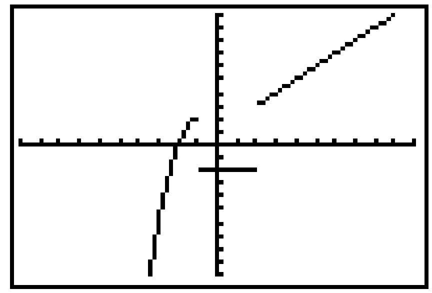
c. $\lim_{x \to -1} h(x)$

d. $\lim_{x \to -1^-} h(x)$

e. $\lim_{x \to 3} h(x)$

f. $\lim_{x \to 3^+} h(x)$

16. Estimate the limits from the given graph.



Note: *Each dot represents 1 unit.*

Be sure to write what each limit is asking for and then estimate its value.

a. $\lim_{x \to 5}$

b. $\lim_{x \to 0}$

c. $\lim_{x \to 2^-}$

d. $\lim_{x \to {}^-2^+}$

e. $\lim_{x \to {}^-1^+}$