Name _____
Date _____

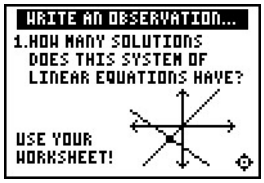
Linear Systems: Using Graphs & Tables — Student Worksheet

Overview

The **Overview** introduces the topics covered in **Observations** and **Activities**. Scroll through the **Overview** using ▶ (◀ to review, if necessary). Read each screen carefully. Look for new terms, definitions, and concepts.

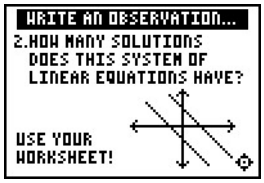
Observations

The **Observations** illustrate mathematical concepts relating to inequalities. Scroll through the **Observations** using ▶ (◀ to review, if necessary). Read each screen carefully. When you come to a **Write an Observation** screen, stop and write the answers to the questions on your worksheet.



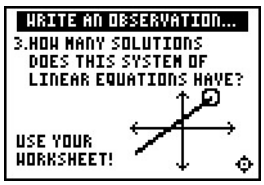
Observation 1

How many solutions does this system of linear equations have?



Observation 2

How many solutions does this system of linear equations have?



Observation 3

How many solutions does this system of linear equations have?

Name _____

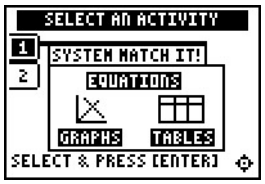
Date _____

Activities

The **Activities** help you practice using graphs and tables. You can select from two different activities—**System Match It!** and a worksheet activity. Follow these steps to play the activities and complete your worksheet.

1. Make sure you are in the **Activities** for this section.

2. Highlight an activity using ▲ or ▼, and press ENTER.



Scoring: You get one attempt to answer the problem. You earn 2 points for a correct answer. You can earn up to 12 points.

Tip: Be careful! The graphs may appear close together.

System Match It!

1. Select the correct answer to the question. Questions include:
   - Selecting the graph that correctly illustrates the system.
   - Selecting the system that correctly describes a graph.
   - Selecting the correct solution of a system given a table.
2. What was your score?

Notes: See TIp™ 4: Creating a Table to help you with tables.

See also Try-It!™ in Chapter 2 Sections 1 and 2 for working with tables and linear equations.

Worksheet Activity

Solve a system using tables.

a. Blue Lake is Pat's favorite place to swim during the summer. There is an entry fee of $12.00 per car per day. Pat notices a sign as they approach the entrance. His family could join the Blue Lake Club for the summer! It costs $48.00 to join and then the entry fee becomes $6.00 per car per day. Fill in the table to help you develop the equations.

Days at the Lake	Cost Without Membership	Cost With Membership
1	$12(1) = $12	$48 + $6(1) = $54
2	$12(2) = $24	$48 + $6(2) = $60
3		
4		
...	...	...
D		

b. Let C = the entry cost to the lake. Let D = the number of days Pat's family goes to the lake during one summer. Write the system of two equations that describe the cost of the entry fee without membership and with membership. Use the table above (in part a) to help you write the equations.

Cost Without Membership: _____

Cost With Membership: _____

Name _____
Date _____

Activities (continued)

c. Check that the equations are written in slope-intercept form, $y = mx + b$. This prepares you to enter the system into your graphing calculator. You will also have to change the variable. Notice that the cost, C, is the dependent variable (Y) and the number of days, D, is the independent variable (X). Write the equations so you can enter them in your graphing calculator.

Y1= _____

Y2= _____

d. Use a table to find the cost of the entry to the lake with and without membership. When is the entry cost without membership equal to the entry cost with membership?

e. If Pat's family goes to the lake 6 times this summer, should they join the Blue Lake Club to save money? Why?

f. If Pat's family goes to the lake 10 times this summer, should they join the Blue Lake Club to save money? Why?

Challenge: Learn how to take a picture of a screen on your graphing calculator using TI Connect™ software and a TI Connectivity cable. You can paste a screen into a word processor and then print it out to hand in for your homework! Go to http://education.ti.com and search for TI Connect™ software.

Name _____
Date _____

Try-It!™ on Your TI-83 Plus or TI-73

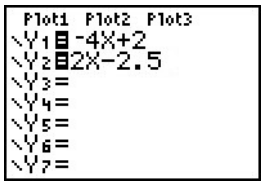
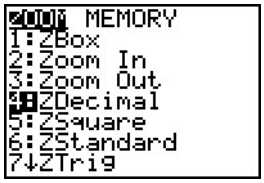
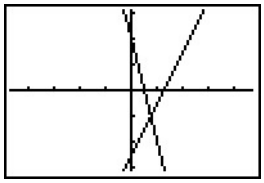
You will:

- Graph two lines of a system of equations.
- Use [TRACE] to locate the intersection of the lines.
- Use [2nd] [TABLE] and [2nd] [TBLSET] to locate the exact solution of the system of equations.

Find the solution of the system: $Y1 = -4X + 2$

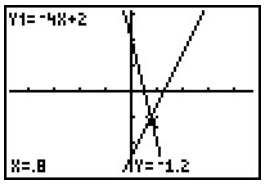
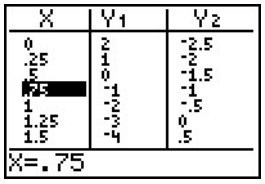
$Y2 = 2X - 2.5$

Notice that these equations are already in the form of y = mx + b. They are in the form to enter into your graphing calculator.

To Do This	Press	Display (TI-83 Plus shown)
1. Exit the Topics in Algebra 1 application and clear the Home screen.	[2nd] [QUIT] ⟨**EXIT**⟩ [CLEAR]	
2. First, enter **-4X + 2** as Y1 and **2X - 2.5** as Y2 in the Y= editor. **Note:** See TIp™ 3: Graphing a Function in the Standard Window for more information. **Note:** On the TI-73, use [x] rather than [X,T,Θ,n].	[Y=] [CLEAR] [(-)] **4** [X,T,Θ,n] [+] **2** [▼] [CLEAR] **2** [X,T,Θ,n] [−] **2** [.] **5**	
3. Select the Zoom Decimal viewing window. The graph displays. **Remember:** You have to adjust the viewing window depending on the system of equations. You can see the intersection of the lines in the ZDecimal window for this example.	TI-83 Plus: [ZOOM] **4:ZDecimal** TI-73: [ZOOM] **8:ZDecimal**	 

Name _____
Date _____

Try-It!™ on Your TI-83 Plus or TI-73 (continued)

To Do This	Press	Display (TI-83 Plus shown)
4. Trace close to the intersection of the graphs to find a value close to the solution. Since Zoom Decimal traces by tenths, you can get close to the answer but this is not the exact answer. Notice that the point (0.8, ⁻1.2) on the screen is on Y1. Is this point on Y2? Check it out! For this particular graphing calculator setup, you don't get the exact answer. Read on to see how to refine the answer.	[TRACE] [▲], [▼], [▶], or [◀]	
5. Use the table setup to refine the solution. Set up your table to show values close to X = 0.8. As shown here, you can use a starting value of 0 with increments of 0.25.	[2nd] [TBLSET] [CLEAR] **0** [▼] **0.25** [CLEAR]	
6. Search through the table to see that the lines intersect at (0.75, ⁻1). Notice that both Y1 and Y2 are ⁻1 when X = 0.75. Verify by hand that the solution is (X, Y) = (0.75, ⁻1). You also know that there is only one solution because the lines intersect at one point, so your search is complete.	[2nd] [TABLE]	

Note: Learn more about the graphing calculator features **intersect** and **Solver** in the Try-It!™ section in Chapter 5: Linear Systems, Section 2: Using Algebra.

Name _____

Date _____

Solution Search

Write the solution and explain how you found the solution using graphs and a table for each of the problems below. Do the following for each problem.

- Rewrite the system in slope-intercept form, $y = mx + b$, if necessary.
- Use the slope-intercept form of the equations to draw a rough sketch of the lines. You can verify your graph on your graphing calculator. Estimate the solution so that you have an idea of how many solutions there are and where the solution is located.
- Search for the solution of the equation on the graphing calculator using graphs and a table.
- See TIp™ 4: Creating a Table and TIp 5: Adjusting the Viewing Window for additional help with the graphing calculator.
- Remember to change your viewing window (WINDOW) or your table setting (2nd [TBLSET]) to do your search.
- Explain how you found the solution.
- Write out how you checked the solution.

Remember: The TI-73 and the TI-83 Plus only use the variables X and Y for graphs and tables. If an equation uses letters other than X and Y, you have to change the variables in the problem to X and Y on the TI-73 and the TI-83 Plus. Use parentheses, if needed, when entering the equations in the Y= editor.

1. $y = 2x + 4$
   $3x + y = -11$

2. $-x + 3y = 4$
   $\frac{1}{3}x + y = \frac{10}{3}$

Name _____
Date _____

Solution Search (continued)

3. $2w + t = 35$
   $\frac{-2}{5}w + \frac{1}{5}t = 19$

4. $4x + 7y = 8$
   $4x + 7y = 14$

5. $x - 9y = 7$
   $2x - 18y = 14$

Linear Systems: Using Graphs & Tables — Teacher Notes

Objectives

- To illustrate how to locate the real number solution of a system of linear equations (two equations and two variables) using tables.
- To illustrate how to locate the real number solution of a system of linear equations (two equations and two variables) using graphs.
- To graphically illustrate the types of solutions expected for a system of linear equations.

Math Highlights

Students work with a system of linear equations that has two equations in two variables. They begin the **Overview** by setting up an analysis of two different cell phone plans. The two plans can be modeled by linear equations. They investigate when the two plans cost the same amount of money.

In the table of values example, students see a table of values for each equation. To create the table, the equations are in the form y = mx + b. They see that the x value that gives the same y value for both equations is the solution. They also see that they may need to refine the table of values to search for the solution.

In the x-y graphical example, students graph both equations and locate the intersection of the lines. The (x,y) coordinate of the intersection of the lines is the solution. Since the graphs of the linear equations in the system can intersect, be parallel, or be the same line, students also see that they may find a unique solution, no solution, or an infinite number of solutions to the system.

In **Observations**, students associate the graphs of the lines of a system with the number of solutions of the system. This is covered again at a higher level in Section 2: Using Algebra.

Common Student Errors

- Students have to rewrite the system in slope-intercept form in order to enter the equations into the graphing calculator. Many students tend to make sign errors and division errors when they rewrite equations. For example, given 2T + 3S = 57 students would first have to rewrite the equation as S = (⁻2/3)T + (57/3), assuming S is the dependent variable. Then, the students have to rewrite this equation as Y1 = (⁻2/3)X + (57/3). A common division error is to write the equation as Y1 = (⁻2/3)X + 57, which is incorrect.
- Students forget to enter fractions into the Y= editor using parentheses. Remind students about the order of operation. If they enter ⁻2/3X, the graphing calculator interprets this as ⁻2 ÷ (3 × X) following the order of operation rules. The correct entry is (⁻2/3) × X.

  Note: TI-73 users can use [b/c] to enter the fractions. However, you should still remind them how to use parentheses and about the order of operation rules.
- When solving by graphing using the graphing calculator, some students trace along one function to what appears to be the intersection point without verifying that that point is also on the other line.

Common Student Errors (continued)

- After students have practiced using graphs and tables to solve a system of equations, they may think that they can always find the exact solution for a system using these methods. Although they can often find exact solutions using these methods, using algebra always gives exact answers for these equations. To help them understand this idea, have students try to search for the solution to the system $y = 2x + 3$ and $y = 2x + 4$. Using a table, they could search forever since these lines are parallel. Using a graph, they might think that the lines are parallel, but they are only looking at a few viewing windows. Ask them how they can know if there is a window where the lines intersect. Open a discussion with your class to see if they think they can verify that this system has no solution using tables or graphs.
- Algebraic methods alone usually do not invite the student to reason out the solution using their knowledge of number sense and geometry. Many students learn the mechanics of solving a problem without understanding the problem or the solution. The graphs and tables method gives students the opportunity to see the values and graphs of the equations so they can see when two equations have the same value.
  Ask students to look at the equations $y = 2x + 3$ and $y = 2x + 4$ again, and use their number sense. Ask them if $2x + 3$ could ever be the same value as $2x + 4$ for a given x? Encourage students to first look at the equations to see if their knowledge of geometry or their number sense can tell them something about the system before they start their method of solution.
- Some visual learners benefit by seeing the numbers and graphs first, and then by using these as the tool to find the solution. However, many students can see the solution to some systems using their number sense. These students may have difficulty taking the time to show and write about their work. This may also be an issue in Chapter 5: Linear Systems, Section 2: Using Algebra. Encourage student to use written mathematics as well as drawing graphs and tables as a communication tool. Have students look in newspapers and on the web for graphs and tables of information to show real examples for the need for this communication skill.

  Require students to write out complete solutions to problems, including the mathematics and the interpretation of what the numbers mean in the problem. For example, the cell phone problem in the **Overview** subsection requires not only the numeric answer but also an explanation about what the numbers mean with respect to the cell phone users.

Student Worksheet Notes with Answers

Overview

Tell students:

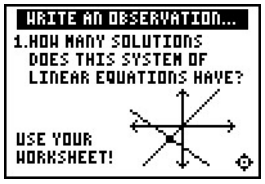
1. How to find the **Overview**, if necessary.
2. How to navigate the application, if necessary.
3. To scroll through the **Overview** on the graphing calculator. Point out new terms, definitions, and concepts, and tell students to look for them as they go through the **Overview**.

Observations

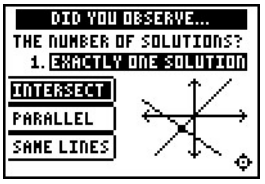
The **Observations** help students start to uncover the types of solutions that arise in systems of linear equations. If necessary, tell students how to find the **Observations** section.

Students are asked to observe the number of solutions from the given graph. They should question whether they are seeing enough of the graph to make a conjecture about the number of solutions.

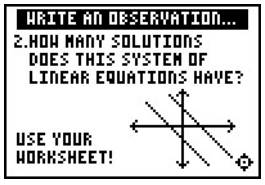
Observation 1

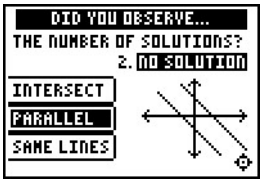


Students see the answer after the third observation question.

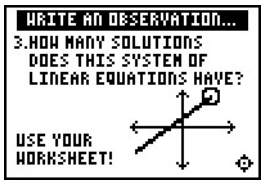


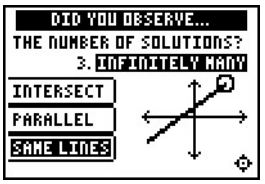
Observation 2



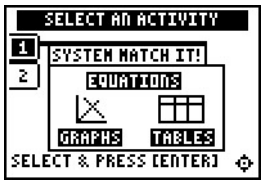


Observation 3





Activities



Scoring: You get two attempts to answer the problem. You earn 2 points for a correct answer on the first try, 1 point for a correct answer on the second try. You can earn up to 12 points.

System Match It!

Tell students to:

1. Select the correct answer to the question. Questions include:
   - Selecting the graph that correctly illustrates the system.
   - Selecting the system that correctly describes a graph.
   - Selecting the correct solution of a system given a table.
2. Record their scores.

Remind students that the graphs might appear very close together on the screen. They need to use their knowledge about both the functions and the graph to determine the correct answer.

Notes: See TIps™ 4: Creating a Table to help you with tables.

See Try-It!™ in Chapter 2 Sections 1 and 2 for working with tables and linear equations.

Worksheet Activity

Students investigate the entry fee to Blue Lake. They compare the entry fees with and without a membership fee.

a. Students should gather information from the problem to write a system of equations for the investigation. Filling in the table with all of the calculations written out helps students develop the equations inductively.

Days at the Lake	Cost Without Membership	Cost With Membership
1	\$12(1) = \$12	\$48 + \$6(1) = \$54
2	\$12(2) = \$24	\$48 + \$6(2) = \$60
3	\$12(3) = \$36	\$48 + \$6(3) = \$66
4	\$12(4) = \$48	\$48 + \$6(2) = \$72
…	…	…
D	12D	48 + 6D

b. Variables are suggested. Review the concept of independent and dependent variables with the students.

Let C = the entry cost to the lake.
Let D = the number of days Pat's family goes to the lake during one summer.

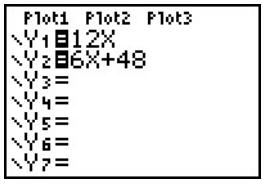
C = 12D
C = 6D + 48 (Students could also enter 48 + 6D.)

Activities (continued)

c. Rewrite the problem in terms of Y1, Y2 and X to prepare to enter the system into the Y= editor.

Y1 = 12X —or— Y1 = 6X + 48
Y2 = 6X + 48 Y2 = 12X

The independent variable (days at the lake) and the dependent variable (entry cost) are stated in the problem. Remind students that the graphing calculator treats X as the independent variable and Y as the dependent variable.

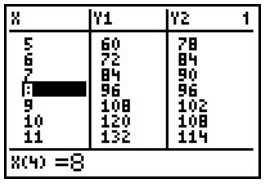


Enter equations in Y= editor.

d. Students need to enter the equations in the Y= editor and should set up the table. Discuss that the domain of the system should be whole numbers starting at 0 since x counts the number of trips to the lake. Notice at x = 8, Y1 = Y2 which is the breakeven point. If Pat's family goes to the lake up to and including 8 times, they might not choose to join the Blue Lake club.

Set up the table.

e. If Pat's family goes to the lake only 6 times during the summer, they will spend more money if they join the club for $48.



Search for where the equations are equal.

f. If Pat's family goes to the lake 10 times during the summer, they will save money if they join the club. They will save $120 – $108 = $16. Discuss the savings if the family goes to the lake more than 10 times. Pose questions such as, when will they save $50?

Have students find other situations that set fees in this manner. One source is the national and state park services web pages.

Try-It!™ on Your TI-83 Plus or TI-73

Students search for the solution of a system of two linear equations in two variables using graphing and tracing, and a table. The problem has been chosen so that students do not trace to the exact solution and need to use the table to search. They could also choose to change the window so that they could trace to the exact solution. This is not an efficient choice, but could be pursued, and the investigation would be enriching.

The students will:

- Graph two lines of a system of equations.
- Use [TRACE] to locate the intersection of the lines.
- Use [2nd] [TBLSET] and [2nd] [TABLE] to locate the exact solution of the system of equations.

Note: Students will learn more about the graphing calculator features intersect (TI-83 Plus) and Solver (TI-83 Plus and TI-73) in the Try-It! section in Chapter 5: Linear Systems, Section 2: Using Algebra.

Tell students to follow the steps exactly on the graphing calculators. Example screens are displayed on the worksheets for students to compare with the graphing calculator screens.

Solution Search

Tell students to:

- Rewrite the system in slope-intercept form, y = mx + b, if necessary.
- Use the slope-intercept form of the equations to draw a rough sketch of the lines. Verify the graphs on the graphing calculator. Estimate the solution so they have an idea of how many solutions there are and where the solution is located.
- Search for the solution of the equation on the graphing calculator using graphs and a table.

  Notes: Since the graphing calculator only uses the variables X and Y for graphs, tables and some other features, students must decide which variable in the problem should be X and which one should be Y when the problem uses other variables. Discuss independent and dependent variables, emphasizing that the graphing calculator is set up to treat X as the independent variable and Y as the dependent variable.

  Remind students to use parentheses correctly when they enter equations into the Y= editor. For example, (1/3)X is not the same as 1/3X which is 1/(3X) when the order of operation rules are applied by the graphing calculator. However, when TI-73 users enter 1/3 using the [b/c] key, their entry is calculated correctly.

  Remind students to change the viewing window ([WINDOW]) or table setting ([2nd] [TBLSET]) to do the search. TIp™ 4: Creating a Table and TIp 5: Adjusting the Viewing Window provide additional help with the graphing calculator.
- Explain how they found the solution.
- Write out the check of their solution.

Answers:

1. (x, y) = (-3, -2)
2. (x, y) = (3, 7/3)
3. (w, t) = (-15, 65) Students are not given which variable is dependent and which is independent. They may very well write the solution as (t, w) = (65, -15).
4. Lines are parallel, which implies that there are no solutions.
5. Lines are the same, which implies that there are an infinite number of solutions.