EXPLORATIONS

Activity 10

Get to the Point! Coordinate Geometry!

Objective

- To find the areas of geoboard polygons using coordinates

Materials

- TI-73
- Student Activity pages (pp. 115 – 117)

In this activity you will

- Use the coordinate method to find the area of geoboard polygons.
- Solve problems using a coordinate approach.

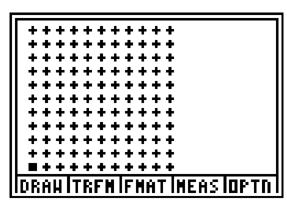
Introduction

The coordinate method for finding area is equivalent to using the surrounding rectangle and subtracting right triangles. This method is good for finding the area of regions that are laid out on coordinates, like those on a map or survey.

Investigation

This investigation will help you find the areas of polygons using coordinates.

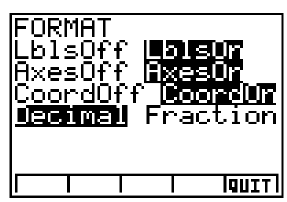
1. From the main Geoboard menu, select **5:11x11**.



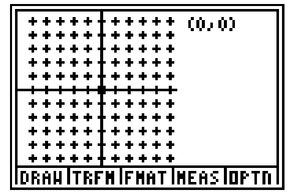
2. To format the geoboard, select **FMAT** and make sure that the following settings are selected:

   LblsOn (Labels are on)
   AxesOn (Axes are on)
   CoordOn (Coordinates are on)
   Decimal (Measurement is in decimal form)

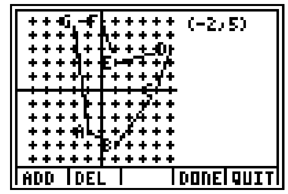
   Select **QUIT** to exit the FORMAT menu.



3. Now change the location of the origin so that the axes intersect in the center of the geoboard. To do this select **OPTN**, **5:Move Origin**, and then press ▶ ▶ ▶ ▶ ▶ ▲ ▲ ▲ ▲ ▲ [ENTER].



4. Next, construct heptagon ABCDEFG by using the points A (-3, -1), B (1, -4), C (4, 0), D (5, 3), E (1, 2), F (0, 5), and G (-2, 5) in this order.



5. To find the area of this or any other polygon when you know the coordinates of the vertices, follow these steps:

a. Starting at any point and moving counterclockwise around the polygon, list the vertices in a vertical column in consecutive order. Repeat the starting point at the bottom of the list.

b. Calculate D, the sum of the down diagonal products.

c. Calculate U, the sum of the up diagonal products.

d. The area of the polygon is one-half of the difference of D and U.

C (4, 0)
D (5, 3)
E (1, 2)
F (0, 5)
G (-2, 5)
A (-3, -1)
B (1, -4)
C (4, 0)

C	**4,**	**0**
D	**5,**	**3**
E	**1,**	**2**
F	**0,**	**5**
G	**-2,**	**5**
A	**-3,**	**-1**
B	**1,**	**4**
C	**4,**	**0**

D = 4(3) + 5(2) +1(5) + 0(5) + (-2)(-1) + (-3)(-4) + 1(0)
= 12 + 10 + 5 + 0 + 2 + 12 + 0
= 41

U = 4(-4) + 1(-1) + (-3)5 + (-2)5 + 0(2) + 1(3) + 5(0)
= -16 – 1 – 15 – 10 + 0 + 3 + 0
= -39

A = ½(D – U)
= ½[41 – (-39)]
= ½(80)
= 40

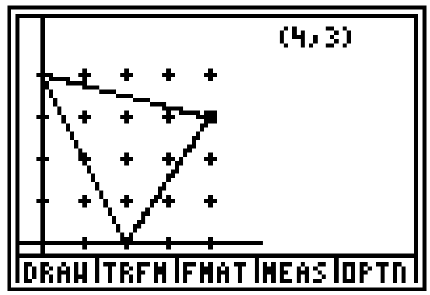
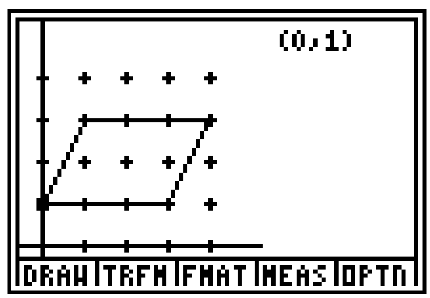
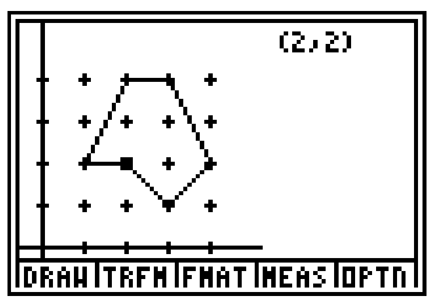
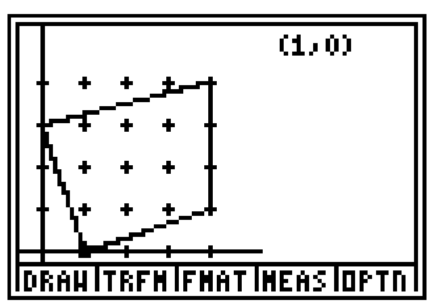
Student Activity

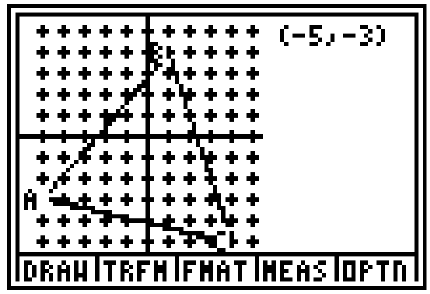
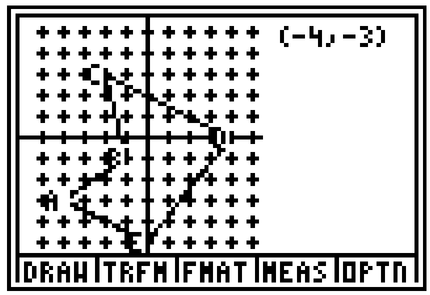
Name ____________________________

Date ____________________________

Activity 10.1: Get to the Point!

Determine the area of each polygon using the Coordinate Method. Check your answers using the TI-73.

1. Area: __________ square units	
2. Area: __________ square units	
3. Area: __________ square units	
4. Area: __________ square units	

5. Area: ___________ square units	
6. Area: ___________ square units	

7. Use the Coordinate Method to find the area of a triangle having vertices at (-10, 5), (17, 20), and (10, -20).

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8. Use the Coordinate Method to find the area of a quadrilateral having vertices at (-10, 0), (0, 20), (30, 0), and (0, -40).

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Student Activity

Name ____________________________

Date ____________________________

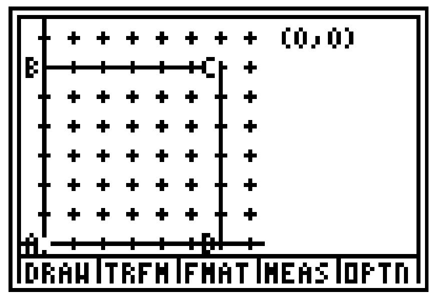
Activity 10.2: Dissected Square

1. Select **3:8x8, FMAT** and make sure that the following settings are selected:

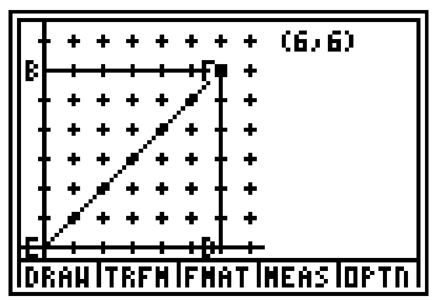
LblsOn (Labels are on)
AxesOn (Axes are on)
CoordOn (Coordinates are on)
Decimal (Measurement is in decimal form)

Select **QUIT** to exit the FORMAT menu.

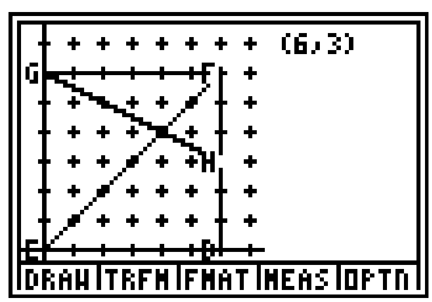
2. Beginning at A (0, 0), construct a square having vertices at A (0, 0), B (0, 6), C (6 , 6), and D (6, 0).



3. Next, draw the diagonal from A (0, 0) to C (6, 6).



4. Now construct the line segment from point B (0, 6) to point H (6, 3), which is the middle of side DF.



5. Now find the ratios of the areas of the four nonoverlapping regions that make up the square. Try to find two different solutions to this problem.

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⑩ The area is 7 square units	⑩ The shape is a concave hexagon
⑩ The shape has 4 right angles	⑩ There are 3 interior points
❿ There are 10 boundary points	❿ The perimeter is $8 + 2\sqrt{2}$ units

Teacher Notes

Activity 10

Get to the Point! Coordinate Geometry

Objective

- To find the areas of geoboard polygons using coordinates

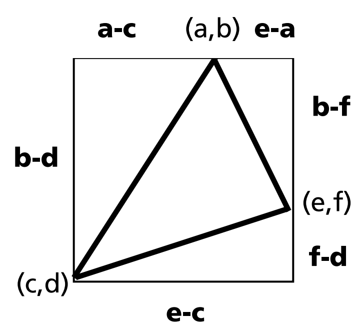
NCTM Standards

- Select and apply techniques and tools to accurately find…area…to appropriate levels of precision
- Use coordinate geometry to represent and examine the properties of geometric shapes

Investigation

If students happen to list the points in a clockwise order when using the coordinate approach, they will get the negative of the correct area.

Suppose we want to find a general formula for the area of a triangle and we know only the coordinates of its vertices (a, b), (c, d), and (e, f). The Coordinate Method is really developed by using the surrounding rectangle approach. Given these coordinates, we can find the lengths of all of the sides of the surrounding rectangle, as shown in the diagram at the right.



Now we can determine the area of the triangle by finding the area of the surrounding rectangle and subtracting the areas of the three right triangles.

Rectangle area – Triangle area – Triangle area – Triangle area

$(b - d)(e - c) - ½(a - c)(b - d) - ½(e - a)(b - f) - ½(f - d)(e - c)$

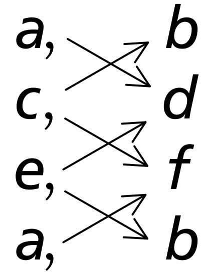
$= be - de - bc + cd - cd + ½bc + ½ad - ½be - ½af + ½de + ½cf$

$= ½be + ½ad + ½cf - ½de - ½bc - ½af$

$= ½[(ad + cf + eb) - (af + ed + cb)]$

This final result is a bit messy and not easy to remember. However, there is a systematic process that easily gives the sums of the two groups shown in parentheses in the final result. This method also generalizes to any number of vertices.

Beginning at any point, write the vertices in counterclockwise order under each other. Repeat the first coordinate at the end. Calculate D, the sum of the down products. Calculate U, the sum of the up products. The area is ½ (D – U), where D = ad + cf + eb and U = af + ed + cb.



For algebra buffs: this final coordinate formula for the area of a triangle can be written as one-half of the sum of three determinants:

$$A = \frac{1}{2}\left(\begin{vmatrix} ab \\ cd \end{vmatrix} + \begin{vmatrix} cd \\ ef \end{vmatrix} + \begin{vmatrix} ef \\ ab \end{vmatrix}\right)$$

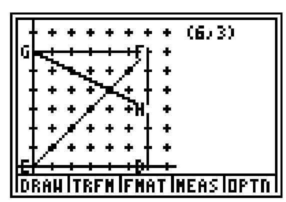
Answers to Student Activity pages

Activity 10.1: Get to the Point!

1. 7 square units
2. 6 square units
3. 5 square units
4. 11 square units
5. 37½ square units
6. 28 square units
7. 487.5 square units
8. 1,200 square units

Activity 10.2: Dissected Square

Method 1: The ratio of areas from smallest to largest is 1:2:4:5.



Since the diagonal EF bisects angle DFG, point X (4, 4) is equidistant from sides FG and DF. Therefore, the altitudes from X to segments FG and DF are equal (in length). Since the two smallest triangles have equal altitudes and sides in the ratio of 1 to 2, their areas are also in the ratio 1 to 2. Now, the smallest and third-largest triangle are similar, with GE being twice the length of FH. Thus, the altitude of the third-largest triangle must also be twice the length of the altitude of the smallest triangle. So, the area of the third largest triangle is 2×2, or 4 times that of the smallest. Finally, a segment XD splits EXHD into two triangles, with EXD having the same area as the third-largest triangle and HXD having the same area as the smallest triangle. The largest region has five times the area of the smallest triangle.

Method 2: Use your TI-73 to find the area of each of the four nonoverlapping regions making up the square.

Group Problem Solving: coordinate geometry

The Group Problem Solving cards are challenge problems that can be used alone or with the individual sections of this book. The problems are designed to be used in groups of four (five or six in a group are possibilities using the additional cards) with each person having one of the first four clues. Students can read the information on their cards to others in the group but all should keep their own cards and not let one person take all the cards and do the work.

The numbers at the top of the cards indicate the lesson with which the card set is associated. The fifth and sixth clues (the optional clues) have the lesson number shown in a black circle.

The group problems can be solved using the first four clues. The fifth and sixth clues can be used as checks for the group's solution or they can be used as additional clues if a group gets stuck. Some problems have more than one solution. Any shape that fits all the clues should be accepted as correct.

One solution for this problem solving exercise: