Cubes and Squares

Teacher Notes & Answers

7 8 9 10 11 **12**

TI-30XPlus MathPrint™ | Activity | Student | 25 min

Teacher Notes:

This is the third activity in the inductive proof series. The first two activities include visual, numerical and algebraic approaches to build relationships for the sum of the first *n* whole numbers (triangular numbers), the sum of the first n-squared whole numbers and the tetrahedral numbers. After students have established the relationships they use proof by induction.

This activity illustrates the remarkable connection between the sum of the first *n* whole numbers and the sum of the first *n* cubed numbers. This is achieved visually, numerically and then proved, once again, by induction.

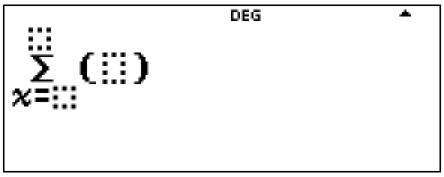
This activity requires the use of the Cubes and Squares slide show.

Calculator Instructions

The first part of this investigation involves summing cubed numbers: $1^3+2^3+3^3+\ldots x^3=\sum_{n=1}^{x} n^3$

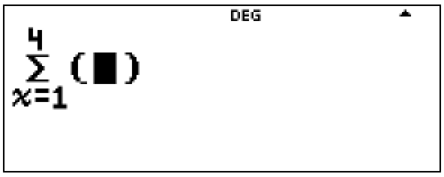
The sum command can be found in the MATH menu.

math | E 5



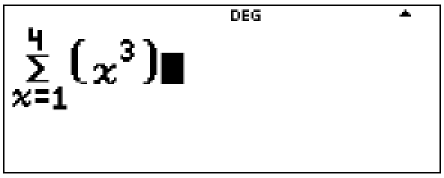
To find the sum of the first 4 numbers cubed enter the numbers 1 and 4 accordingly:

A 1 | ▲ | D 4 | ►



The numbers need to be cubed **before** they are added.

clear var x^{yzt}_{abcd} | $x^{\square}$ | C 3 | ► | ►



Press enter to determine the result.

Question: 1.

Determine the sum of the first 10 numbers cubed: $1^3 + 2^3 + 3^3 + \ldots 10^3$.

Answer: $1^3 + 2^3 + 3^3 + \ldots 10^3 = 1 + 8 + 27 + \ldots 1000 = 3025$. [The calculator instructions show a quicker method]

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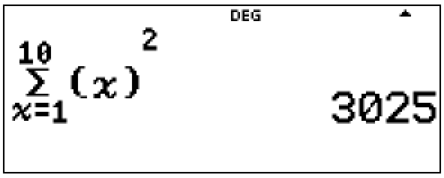
Question: 2.

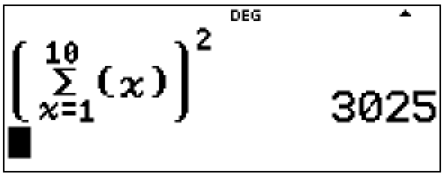
Square the sum of the first 10 whole numbers and comment on the result: $(1+2+3+...10)^2 = \left(\sum_{n=1}^{x} n\right)^2$

Answer: $(1 + 2 + 3 + \ldots 10)^2 = 55^2 = 3025$. The calculator instructions provide a quicker method, students must remember to place the squared sign outside the summation computation. Note that the answer is the same as Question 1.

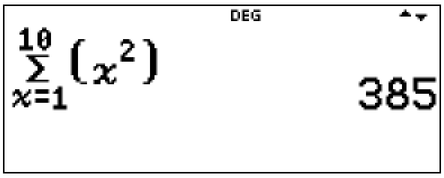
Correct:



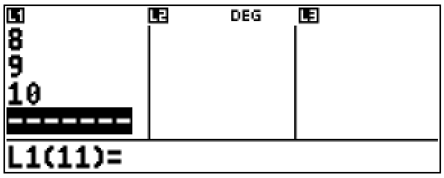
OR



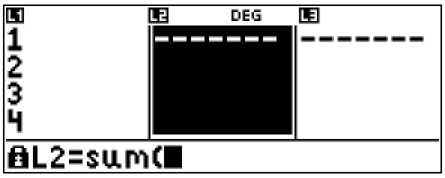
Incorrect:



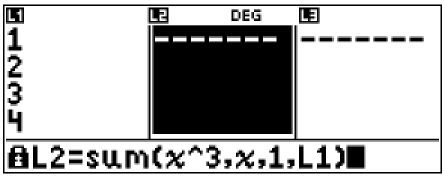
Enter the numbers 1 to 10 in List 1.



Navigate across to List 2 and enter the sum formula:

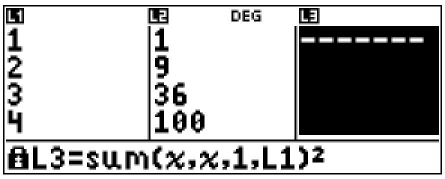


The syntax for the sum command in this environment is as follows: sum(*expression*, *variable*, *start*, *end*)



Then press:  to execute the calculations.

List 3 needs to have a formula for the squared sum of whole numbers. The formula should entered as shown opposite:



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Question: 3.

Complete the following table of values:

N	1	2	3	4	5	6	7	8	9	10
N^3	1	8	27	64	125	216	343	256	729	1000
$\sum_{x=1}^{n} x^3$	1	9	36	100	225	441	784	1296	2025	3025
$\sum_{x=1}^{n} x$	1	3	6	10	15	21	28	36	45	55
$\left(\sum_{x=1}^{n} x\right)^2$	1	9	36	100	225	441	784	1296	2025	3025

Answer: Note that $\sum_{x=1}^{n} x^3 = \left(\sum_{x=1}^{n} x\right)^2$ for all values x = {1, 2, 3 ...10}

Question: 4.

Write down the fomula for $\sum_{x=1}^{n} x$ and hence the formula for $\sum_{x=1}^{n} x^3$.

Answer: $\sum_{x=1}^{n} x = \frac{x(x+1)}{2}$ and $\sum_{x=1}^{n} x^3 = \frac{x^2(x+1)^2}{4}$ based on observations from the table.

Question: 5.

Use induction to prove the formula for the sum of the first n^3 whole numbers.

Answer:

Required to show that $\sum_{n=1}^{x} \left(n^3\right) = \frac{x^2(x+1)^2}{4}$

Step 1: Show true for $x = 1$

LHS: $\sum_{n=1}^{1} \left(n^3\right) = 1$

RHS: $\frac{x^2(x+1)^2}{4} = \frac{1 \times 2^2}{4} = 1$

Step 2: Assume true for x

That is: $\sum_{n=1}^{x} \left(n^3\right) = \frac{x^2(x+1)^2}{4}$ **-- Equation 1**

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Step 3: Show true for $x + 1$.

Working with the LHS

$$\sum_{n=1}^{x+1}\left(n^3\right)=\sum_{n=1}^{x}\left(n^3\right)+(x+1)^3$$
$$=\frac{x^2(x+1)^2}{4}+\frac{4(x+1)^3}{4}$$
$$=\frac{(x+1)^2(x^2+4(x+1))}{4}$$
$$=\frac{(x+1)^2(x^2+4x+4)}{4}$$
$$=\frac{(x+1)^2(x+2)^2}{4}$$

Working with the RHS

$$\frac{(x+1)^2(x+1+1)^2}{2^2}=\frac{(x+1)^2(x+2)^2}{4}$$ replace x with $x+1$

$\therefore$ LHS = RHS

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