Properties of Trapezoids and Isosceles Trapezoids

ID: 9084

Time required
25 minutes

Activity Overview

A trapezoid is a quadrilateral where one pair of sides is parallel while the other two sides are not. In an isosceles trapezoid the non-parallel sides are congruent. In this activity we will attempt to create an isosceles trapezoid from an ordinary trapezoid, then approach the problem in a different manner and finally, examine the properties of trapezoids.

Topic: Quadrilaterals & General Polygons

- *Prove and apply theorems about the properties of rhombuses, kites and trapezoids.*

Teacher Preparation and Notes

- *This activity is designed to be used in a middle-school or high-school geometry classroom.*
- *This activity is intended to be mainly* ***teacher-led****, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their graphing calculators.*
- *For this activity, students should know the definitions of a trapezoid and isosceles trapezoid. If they do not already know these terms, you can define them as they appear in the lesson, but allow extra time to do so.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9084" in the keyword search box.***

Associated Materials

- *PropTrapezoids_Student.doc*

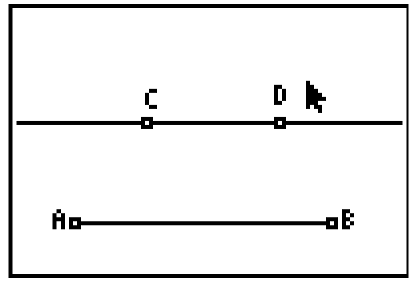
Suggested Related Activities

To download any activity listed, go to *education.ti.com/exchange* *and enter the number in the keyword search box.*

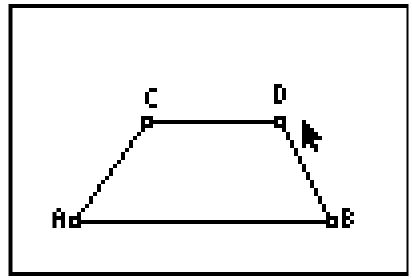
- *Rhombi, Kites, and Trapezoids (TI-84 Plus family) — 12093*
- *Creating a Parallelogram (TI-Nspire technology) — 11987*

To begin, students should start with a blank Cabri Jr. document. They should construct $\overline{AB}$ and a point *C* above the segment.

Create a line through *C* that is parallel to $\overline{AB}$. Construct a point on the new line and label it *D*.

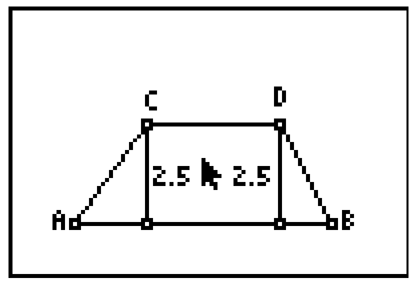


Hide the parallel line and complete the trapezoid. Measure the interior angles. Base angles at *C* and *D* are not congruent, but they can be. Will they ever be? Base angles at *A* and *B* will be congruent if angles *C* and *D* are congruent. The lengths of the diagonals $\overline{AD}$ and $\overline{BC}$ would be congruent if *C* and *D* are congruent.

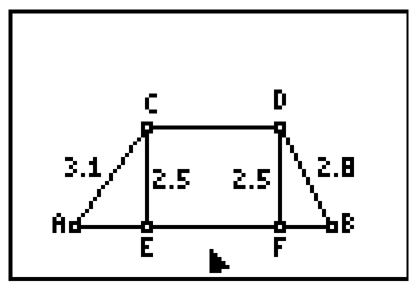


To verify that the lines are parallel, students construct lines through *C* and *D* that are perpendicular to $\overline{AB}$.

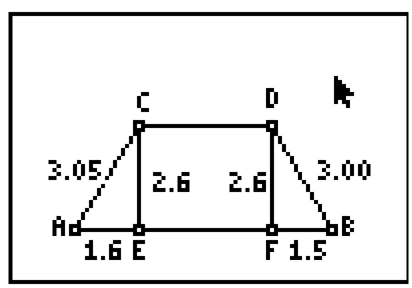
Then construct the points of intersection of the new lines with $\overline{AB}$. Construct lines segments to connect *C* and *D* to the line through $\overline{AB}$ and measure the lengths of these segments. These segments should always be equal because side *AB* is parallel to side *CD*.



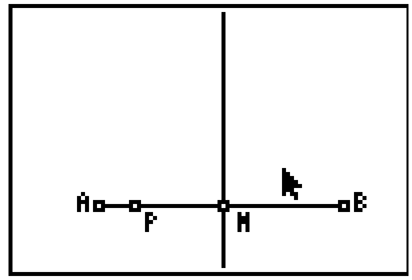
Label the new points on $\overline{AB}$ as *E* and *F* and measure the lenghts of $\overline{AC}$ and $\overline{BD}$. Line segments *AC* and *BD* will be congruent when base angles *C* and *D* are congruent.



Try to drag point *C* or point *D* to make *AC* = *BD*. Due to the screen resolution, this can be very difficult. Construct $\overline{AE}$ and $\overline{BF}$. For an isosceles trapezoid, these segments should also be congruent because triangles *AEC* and *BFD* are congruent due to the side-angle-side property.

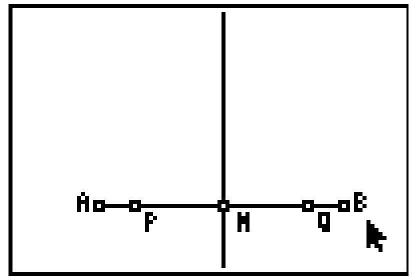


In order to construct an isosceles trapezoid, start with a line segment, *AB*. Construct the midpoint, *M*, and another point, *P*, on $\overline{AB}$. Construct a line through the *M* that is perpendicular to $\overline{AB}$.

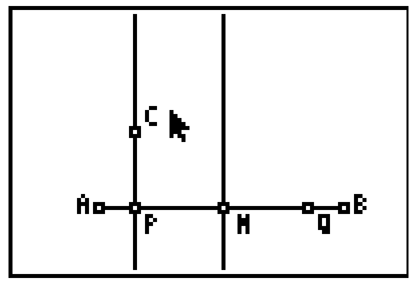


Press TRACE and select the Reflection option. Click on the perpendicular line through *M* and then point *P*.

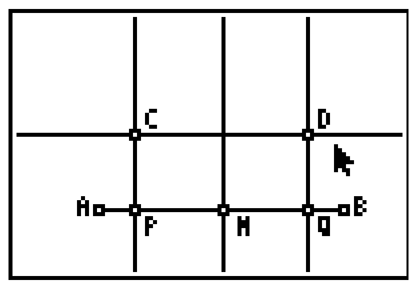
A new point will appear on the line segment on the right side. Label this point *Q*.



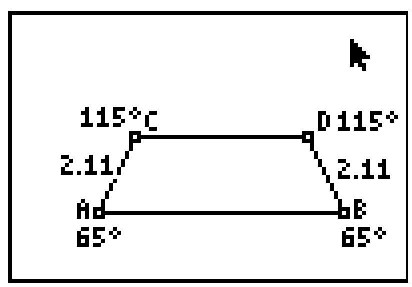
Construct perpendicular lines through *P* and *Q*. Construct point *C* on the perpendicular through *P*.



Construct a perpendicular to $\overline{AB}$ through point *Q* and a line through *C* that is parallel to $\overline{AB}$. Construct point *D* at the intersection of these two lines.



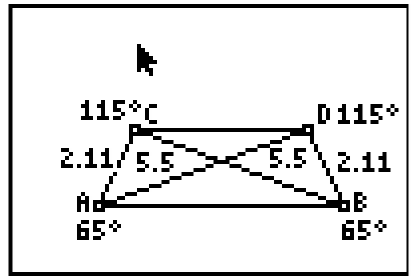
Hide the parallel and perpendicular lines and points *M*, *P* and *Q*. Complete the trapezoid and measure $\overline{AC}$ and $\overline{BD}$. In an isosceles trapezoid, the diagonals are congruent and the adjacent base angles are congruent.



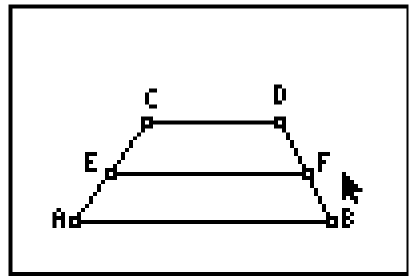
Measure the base angles—the interior angles at *A*, *B*, *C* and *D*. Angles *C* and *D* are congruent and angles *A* and *B* are congruent.

Construct and measure the lengths of the two diagonals $\overline{AD}$ and $\overline{BC}$. They are congruent because the sets of base angles are also congruent.

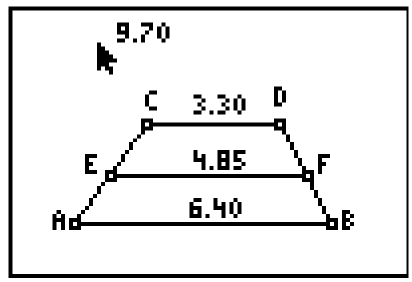
Drag point *C* and watch the angles and sides. All of the above properties are preserved.



However, one property that is common to all trapezoids is that the line segment connecting the non-parallel sides is also parallel to these sides and its length is half the sum of the parallel sides. Construct a trapezoid *ABCD*. Construct the midpoints at *E* and *F* and construct $\overline{EF}$.



Measure the lengths of $\overline{CD}$, $\overline{EF}$ and $\overline{AB}$. You could prove that $\overline{EF}$ is parallel to $\overline{CD}$ and $\overline{AB}$ by calculating



Use the Calculate tool to find the sum of the lengths of $\overline{CD}$ and $\overline{AB}$.

Place the number “2” on the screen and divide the sum by 2 using the calculate tool. This result always equal the length of $\overline{EF}$.